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Seismic Refraction Shooting in an Area of the Eastern Atlantic

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SEISMIC REFRACTION SHOOTING IN AN AREA OF THE EASTERN ATLANTIC

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CONTENTS

For the experiments described in this paper a new method of seismic refraction shooting was developed. With this method hydrophones suspended at a depth of about 100 ft. below the surface of the sea acted as receivers for the compressional waves developed by depth charges exploding at a depth of approximately 900 ft.

The hydrophones were connected with sono-radio buoys which radio-transmitted the electrical signals to a recording system in the ship from which the charges were dropped. Four buoys were in use simultaneously, distributed at differing ranges from the ship.

The experiments were carried out at three positions in an area of the eastern Atlantic around the point 53° 50′ N, 18° 40′ W, where the water depth is approximately 1300 fm. (2400 m). The results showed that the uncrystalline sedimentary layer in this area varied in thickness from 6200 ft. to 9700 ft. (1900 to 3000 m), and that the velocity of compressional waves in it increased from the value for sea water, 4900 ft./s (1·5 km/s), at the surface with an approximately constant gradient of 2·5/s to a limiting value of 8200 ft./s (2·5 km/s). Below the sedimentary layer there was a crystalline rock with compressional wave velocity of approximately 16500 ft./s (5·0 km/s) and of thickness varying between 8800 ft. (2700 m) and 11100 ft. (3400 m). The base of this layer was in both determinations at approximately 25500 ft. (7800 m) below sea-level. The lowest layer concerning which information was obtained gave a value for the compressional wave velocity of about 20500 ft./s (6·3 km/s), but was of undetermined thickness.

The characteristics of the sedimentary layer were such as might be expected for a continuous succession of deep-sea sediments, the thickness on this basis being such as to indicate the long existence of the ocean in this area. On the other hand, it is possible that it represents a down-warped continental shelf. The layer below the sedimentary layer has a compressional wave velocity which is low for an igneous rock at this depth, and it is probable that it represents a crystalline sedimentary rock. From the evidence it is not possible to determine whether this rock is of continental or deep-sea origin. The lowest layer of these experiments is unlikely to have a constitution similar to that of the European granitic layer, since the compressional wave velocity in it would, on this hypothesis, be exceptionally high. The value is, however, close to that calculated by Jeffreys for the intermediate layer.

1. Introduction

The work described below originated from a suggestion made by Bullard & Gaskell (1941) concerning the possibility of the use of hydrophones rather than geophones as receivers for seismic refraction shooting at sea. Since it was unnecessary for these hydrophones to be placed on the sea bed (Hill & Willmore 1947), there was no limitation to the depth of water in which these experiments could be undertaken, and as a result there has been an extension of the refraction shooting method to the deep oceans where the problems are extensive and the knowledge, so far, trivial. Ewing *et al.* (1950*a*) has independently developed similar methods.

A further requirement in the development has been that only one ship should be used. Primarily this was desirable on account of the expense of the two-ship technique, but, secondly, by the use of several sono-radio buoys more information can be obtained from a single shot than with the other method.

The deep-sea experiments were unfortunately confined to a single area in the Atlantic which was perhaps not representative of the deep water, the depth being only about 1300 fm. (2400 m). This limitation of area was forced upon us, since we were unable to obtain a ship which could be employed solely upon our experiments and it was therefore necessary for us to work from one of the British weather ships while on a routine tour of duty. This expedition served, nevertheless, to show the possibilities of the method and provided detailed information concerning the structure of the area.

During the expedition *reflexion* shooting experiments were carried out, but the subbottom echoes obtained in the area were confused, any one record showing several apparent reflexions which, in general, were unrepeatable. With the layering determined from the *refraction* shooting experiments it would not be expected that the reflecting horizons in the sea bed would be at a sufficiently small depth to be apparent on the reflexion shooting records, and therefore in a negative sense these experiments helped to confirm the interpretation of the refraction shooting results. The technique which was used was similar to that described by Hersey & Ewing (1949). Since no positive results were obtained from this method and since the technique was not novel, the numerous records are not described in detail in this paper.

During the experiments and the preliminary analysis of the results, the work was the joint responsibility of the writer and Mr J. C. Swallow. Owing to his absence abroad the latter was unable to share in the preparation of this paper and for this reason only has it been written under the name of one author.

2. General description of apparatus for refraction shooting

A schematic diagram of the layout of the equipment for receiving and recording the pressure waves in the water resulting from the explosion of the depth charge is shown in figure 1.

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A pressure-sensitive crystal hydrophone is hung below a buoy at a depth of about 150 ft. The buoyancy of the hydrophone is made slightly positive by the addition of a float. This causes the cable connecting it to the buoy to hang in a bight, protecting the hydrophone against the vertical motion of the buoy. The output from this hydrophone passes along a cable to the main amplifier in the buoy; this amplifier is designed to have characteristics

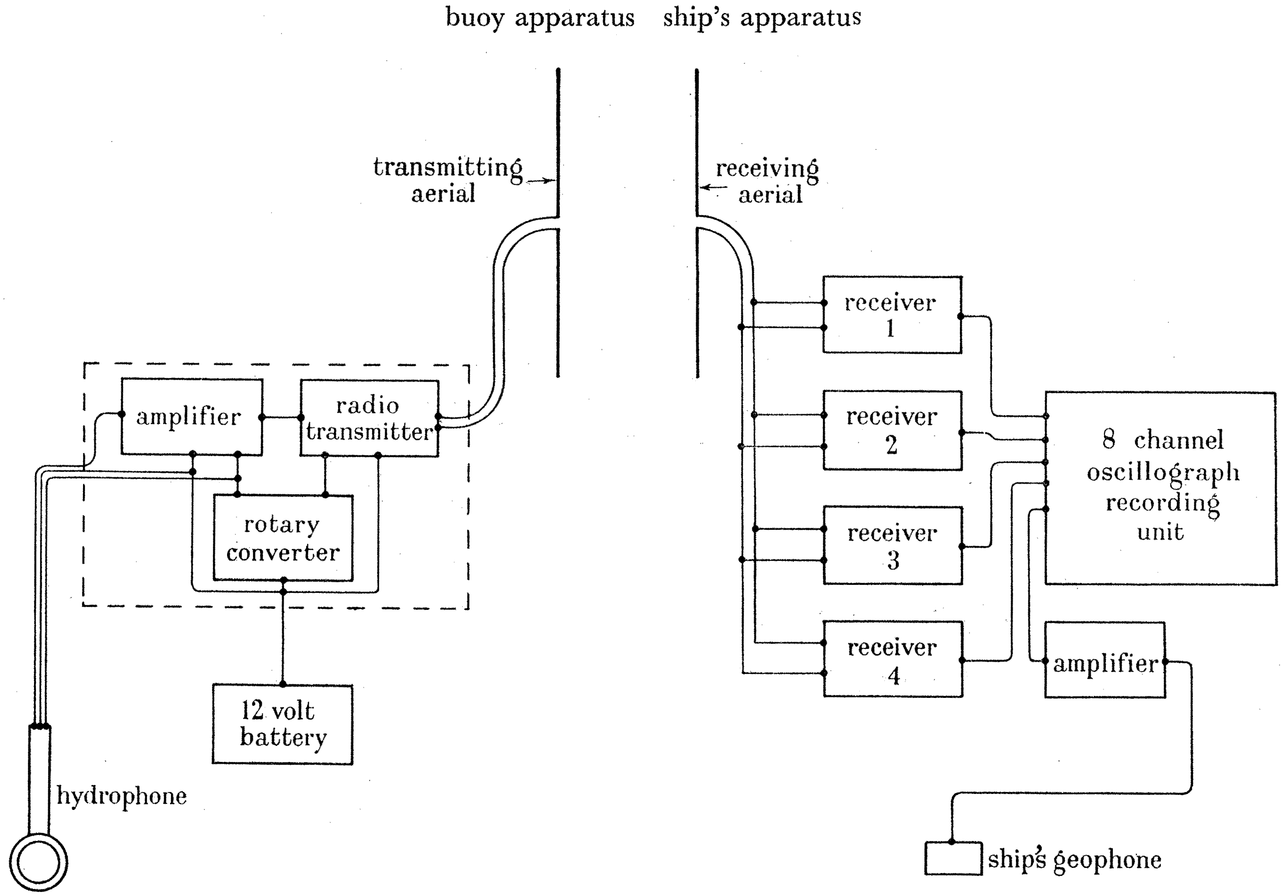

FIGURE 1. Schematic diagram of apparatus in buoy and ship.

such that the peak level of the output is linear with respect to the logarithm of the peak level of the input signal over a range of input signal levels of about 500 to 1. The output from the amplifier is connected to the frequency modulator of a radio transmitter working at a frequency about 42 Mc/s. Frequency modulation is preferable to amplitude modulation, since it eliminates spurious signals which may otherwise be produced by movements of the buoy aerial. The power from the transmitter is radiated from a vertical dipole aerial mounted with centre about 10 ft. above the buoy. The supply for the electrical apparatus in the buoy is provided by a 12 V car accumulator contained in a water-tight box attached to the base of the bottom pole of the buoy, the weight of the battery and its box maintaining the buoy vertical. The high-voltage supply for the amplifier and transmitter is obtained from a rotary converter running off the 12 V supply. A maximum of four buoys are used simultaneously, the transmitter of each being tuned to operate on a separate radio-frequency channel.

The radio transmissions from the buoys are picked up in the ship by receivers, the outputs from which pass to single-stage amplifiers and thence to electro-mechanical oscillographs. The oscillograph deflexions are recorded photographically on moving paper. Timing marks are provided by an interrupted light beam controlled by a tuning fork and phonic motor.

The output from a small moving-coil geophone placed against the inside of the ship's hull below the water-line is connected to another oscillograph. The deflexions of this oscillograph give the instant of explosion, the depth of the water and the depth of the explosion. Standard Naval depth charges containing 200 lb of amatol and set to fire at 900 ft. were used as the explosive sound source.

A diagrammatic sketch of one of the sono-radio buoys is shown in figure 2.

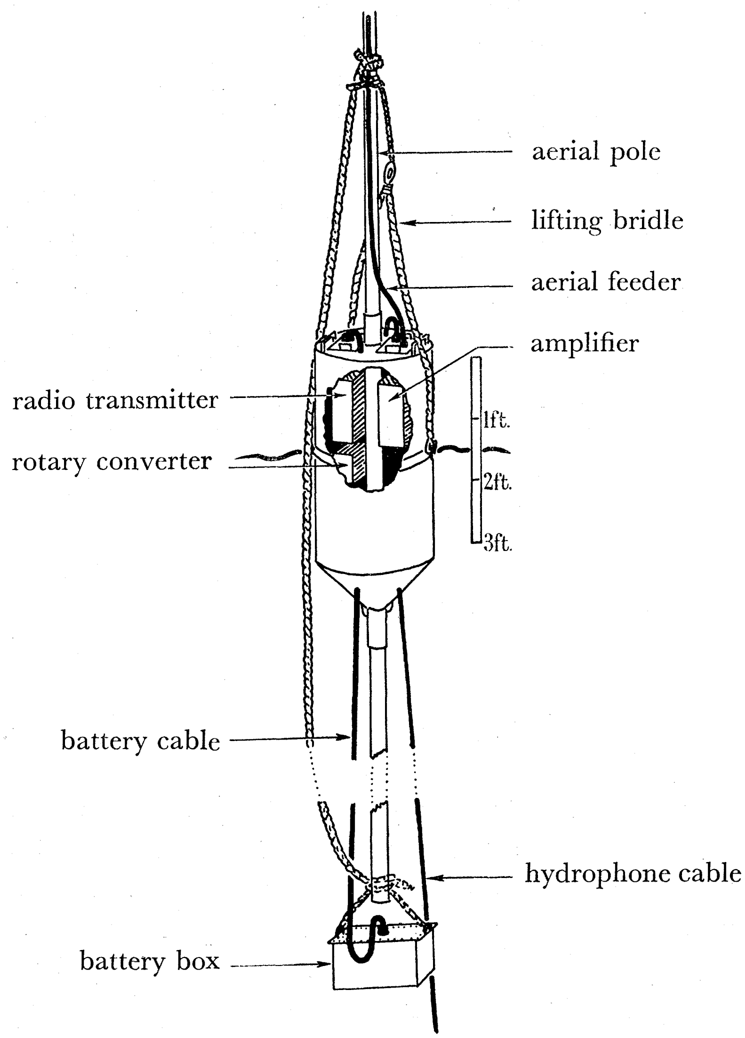

FIGURE 2. Sono-radio buoy.

3. EXPERIMENTAL METHOD

3·1. *Narrative*

The experiments were carried out from the Ocean Weather Ship *Weather Explorer* on her routine meteorological voyage of August 1949. The station she was occupying on this voyage, which was of four weeks' duration, was that called 'J' positioned at lat. 53° 50′ N,

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long. 18° 40′ W. The ship was restricted to an area of approximately 30 miles radius centred at this position. In this area the water depth is approximately 1300 fm. (2400 m), but shoals to about 1000 fm. (1800 m) in the northern part and deepens to about 1400 fm. (2600 m) to the south. The position of the station is shown in figure 3.

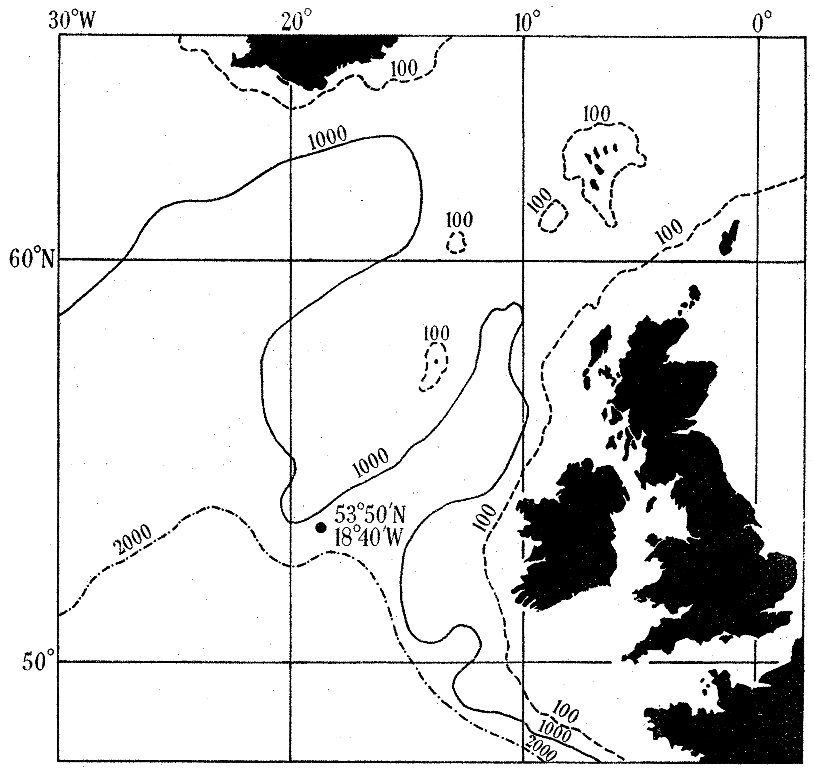

FIGURE 3. Small-scale chart of area. (Soundings in fathoms.)

3·2. *Surveying the area*

Throughout the voyage, opportunities were taken whenever possible to obtain soundings. These were principally obtained with the ship's echo-sounder, but were also obtained from the depth charges and from the reflexion shooting shots. The echo-sounder was not designed for depths of water greater than a few hundred fathoms, but an amplifier installed for the expedition allowed the sensitivity to be increased sufficiently to enable good reflexions to be obtained in the deep water provided the ship was stopped. With the ship under way no reflexions could be received on account of the background noise, and this considerably limited the information which was obtained.

The soundings obtained are shown in the large-scale chart of the area drawn in figure 4. The two soundings underlined were obtained from the Admiralty chart of the area. This chart also shows the position of the refraction shooting lines. The comparative accuracy of the soundings as obtained from the echo-sounder was approximately ±3 fm. (±6 m), but the absolute accuracy was not better than ±15 fm. (±30 m) on account of our inadequate knowledge of the variation of velocity with depth below the surface of the sea. The values for the velocity were obtained from tables published by the Admiralty (1939).

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The position fixing was carried out by celestial observations, and also by the radio methods Consol and Loran. Neither of these radio methods provided a good coverage of the area and could not be relied upon to an accuracy of better than 5 miles; this meant that by day the accurate position fixing depended upon sun sights and resulted in the possibility in overcast weather of considerable errors.

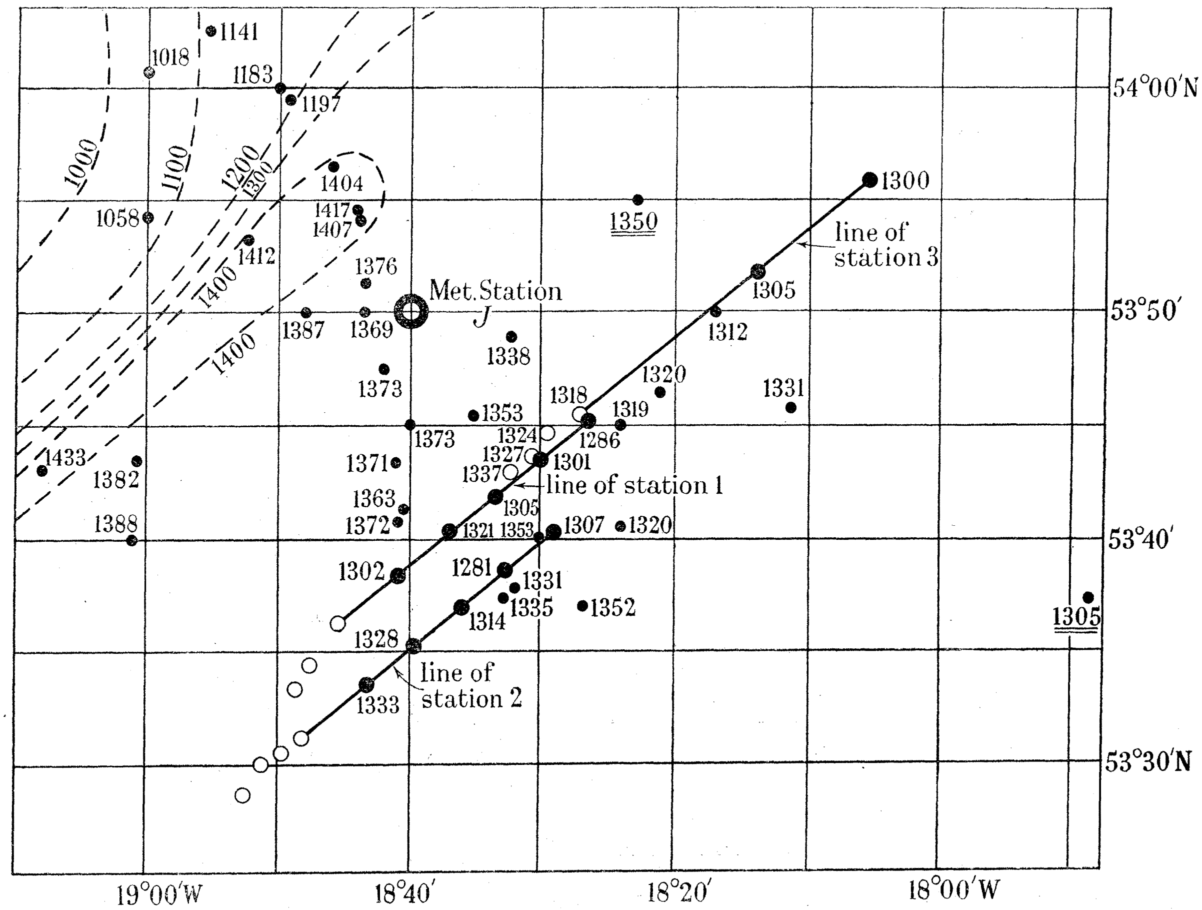

FIGURE 4. Chart of area. ○, buoy positions; ●, shot positions. (Soundings in fathoms.)

3·3. *Buoy launching and positioning*

The buoys were launched along a line in the direction in which the charges were to be dropped, and the spacing between them was roughly $\frac{3}{4}$ sea mile, accurate spacing within this short distance being impracticable, since it was desirable to launch the buoys in the shortest possible time on account of the limited life of their batteries.

The direction of the lines along which the refraction shooting experiments were carried out was selected in order that there should be the minimum change in depth along the line. The position was chosen in order that, within the limits of the area, the water depth should be as great as possible.

3·4. *Dropping the depth charges and recording the signals*

After the last buoy had been dropped the ship was worked up to a speed of about 9 knots and steamed a minimum distance of 6 sea miles before the first charge was dropped.

Thereafter the charges were dropped at equal intervals of time, with the ship proceeding at a constant speed. The splash of the depth charge hitting the water was a signal for a stop-watch to be started. The sinking rate and firing depth were approximately 10 ft./s (3 m/s) and 900 ft. (270 m) respectively, and approximately 90 s therefore elapsed between when the charge was dropped and when it detonated.

The recording camera was switched on approximately 60 s after the release of the charge, and at the same instant the ship's main engine was stopped in order to limit the amount of vibration picked up on the internal geophone.

The 30 s of recording time before the charges fired was allowed for two reasons. First, it was most important that the depth-charge firing signal picked up by the ship's internal geophone should be recorded, and by allowing a considerable margin of time there was little danger that this should fail to be recorded if the depth charge fired shallower than the expected depth. Secondly, it is desirable to have a length of record from which the buoy hydrophone, and radio transmitter, background level can be determined. After the explosion, the camera was left switched on until the oscillographs had reached background level following the arrival of the direct sound through the water. For the long-range shots the total recording time was approximately 1 min, which with a paper speed of 4 cm/s, resulted in a record about $2\frac{1}{2}$ m in length. At this paper speed the arrival times could be read to an accuracy of about 0·005 s.

Immediately the recording was complete the ship proceeded to the next station, usually 3 sea miles farther on. The time taken to reach the position was adequate for the development, fixing and inspection of the record.

4. Experimental results

4·1. *Characteristics of records*

A foreshortened tracing of the three records obtained from a single shot on three of the sono-radio buoys is shown in figure 5. The arrival of the sound on the ship's internal geophone is shown at the point *A*, and the bottom reflexion at the point *B*. The path representing the sound reflected first from the sea surface and then from the sea bed is at the point *C*. The point *D* represents the second reflexion from the sea bed. The ground-wave arrivals are marked *E*, and are followed by continuous trains of waves of not greatly varying amplitude, and from which attempts to read other arrivals which can be correlated between different shots and buoys have not been consistently successful. At the points marked *F* the direct sound through the water arrives and this is followed by pulses *G*, of great amplitude, representing the ray which has been refracted on a curved path in the top layer of the sea bed where there is a gradient of velocity. The subsequent pulses, which are again of large amplitude, are the rays which have been refracted in the sea bed and then reflected from the surface of the sea progressively greater numbers of times. The times of arrival of the rays which have been reflected from the sea bed and surface are not visible. The number of the multiple refractions is limited for any one range between shot point and receiver, an observation which is of importance in confirming that these arrivals represent refractions rather than reflexions.

Attempts have been made with the records to correlate the frequencies present at various points in the ground waves with the track followed by the arrivals, but there was not good evidence for assigning definite frequencies to arrivals from given layers. The main frequency of that part of the wave train preceding the arrival of the direct sound through the water was between 8 and 12 c/s. The arrival of the direct sound was characterized by a sudden increase in both amplitude and frequency, the maximum frequency apparent on the record being the upper limit of the frequency passband of the buoy amplifiers, 200 c/s. The multiple refractions following the direct sound through the water contained lower frequencies than those in the direct sound except for a short-duration high-frequency component at the beginning of the pulse. The main frequency decreased with increasing order of the refraction. Thus for a shot at 15 sea miles the first of the multiple refractions contained a main frequency of 25 c/s, the second approximately 20 c/s, the third 17 c/s the fourth 13 c/s and the fifth 10 c/s.

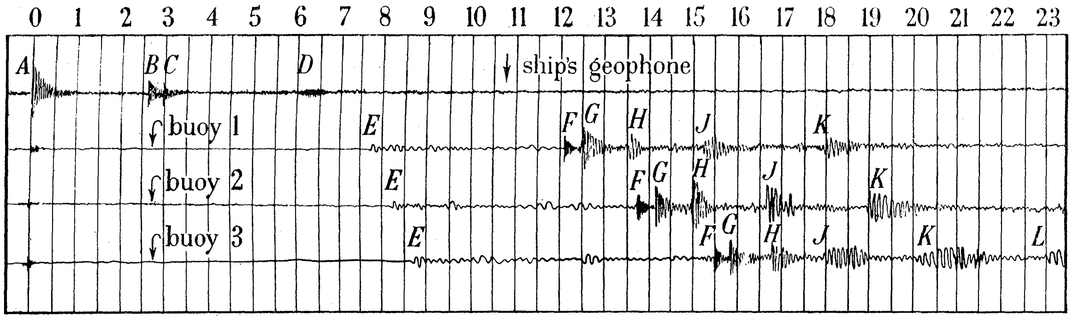

FIGURE 5. Foreshortened tracing from a single shot. *A*, arrival of sound at ship; *B*, bottom reflexion; *C*, surface-bottom reflexion; *D*, second bottom reflexion; *E*, ground wave arrival; *F*, direct sound; *G*, zero-order multiple refraction; *H*, *J*, *K*, *L*, first, second, third, fourth order multiple refractions.

The low-frequency cut-off of the system was set by the hydrophone depth (Press & Ewing 1950) and would depend upon the angle of inclination of the normal to the wave front. The lowest frequency which could pass through the system without distortion was about 10 c/s and occurred when the angle of inclination was zero. For the multiple refractions this angle will decrease with the increasing order number, and for this reason alone it is to be expected that there will be a decrease in the frequency with increasing order. Similarly, for the direct sound through the water, where the angle of inclination will be approximately 90°, the lower frequencies would not appear. Since therefore it would seem that the main frequency components of the wave trains could be equated to the frequency limits of the receiving system it is not considered that detailed analysis would be profitable.

It was expected that the bubble oscillations (Cole 1948) from the explosion might obscure the details of the arrivals, but, although these oscillations have been sought from the records, they have in no case been found.

4·2. *Corrections to observed times of arrival of ground waves*

The times of arrival of the ground waves obtained directly from the records are relative to the signal received by the ship's internal geophone from the explosion. This does not represent the actual time of explosion on account of the depth at which the charge fires and the horizontal distance the ship has moved between when the charge was dropped and when it fired. To correct the times of arrival to the instant of detonation rather than the instant when the shock was felt in the ship, it is therefore necessary to calculate the slant distance between the ship's internal geophone and the point of detonation. This calculation involves the knowledge of (*a*) the ship's speed, (*b*) the time between dropping the charge and its explosion and (*c*) the depth of firing of the charge. The absolute accuracy which is desirable in the determination of the slant range is approximately ±100 ft. (±30 m), while the relative accuracy between successive shots could with advantage be ±50 ft. (±15 m). This degree of accuracy requires that the ship's speed be absolutely known to within ±5 % and relatively known to ±3 %, while the figures for the firing depth should be respectively ±8 % and ±5 %; these values assume that the time between the dropping and firing of the charge can be measured with a much higher percentage accuracy.

It was expected that the ship's speed would be known with precision from the number of revolutions per minute of the engine. This was not, however, the case, and it was found that a better determination could be obtained by measurement of the distance moved and the time between successive shots. The distance measurement was obtainable from the time taken for the direct sound to reach the buoy hydrophones, while the time between successive shots was measured from the chronometer. The main error in determining the absolute value of the ship's speed arose from the fact that there was a relative drift between the buoys, but it is estimated that it does not exceed 5 %. The timing between the charge hitting the water and its explosion was obtained with a stop-watch to an accuracy of about 0·5 %.

The ship's internal geophone not only provided an indication on the recording when the sound arrived at the ship, but also provided the times of various bottom and surface reflexions. The paths by which these sound pulses arrived are as follows:

(*a*) The direct sound.
(*b*) The sound reflected once from the sea bed.
(*c*) The sound reflected once from the surface and once from the sea bed.
(*d*) The sound reflected twice from the sea bed and once from the surface.

From the differences in the times of arrival between paths (*a*) and (*b*) and paths (*a*) and (*c*) it is possible to determine the depth of water beneath the charge, and also the depth at which it fired.

Let t_1 be the time difference between paths (*a*) and (*b*),
t_2 be the time difference between paths (*a*) and (*c*),
h the depth at which the charge fired,
d the depth of water,
x the horizontal distance between the ship's internal geophone and the vertical through the point of detonation,
R the slant range between the point of detonation and the geophone,
V_0 the mean vertical velocity of the sound in the water,
V_1 the horizontal velocity of the sound in the water,

then

$$\left.\begin{aligned} t_1 &= \left\{\frac{(2d-h)^2+x^2}{V_0^2}\right\}^{\frac{1}{2}} - \frac{R}{v_1}, \\ t_2 &= \left\{\frac{(2d+h)^2+x^2}{V_0^2}\right\}^{\frac{1}{2}} - \frac{R}{v_1}. \end{aligned}\right\} \quad (1)$$

Expanding both these expressions and neglecting terms in $\dfrac{x^2}{(2d\pm h)^2}$ of higher order than the first it can be shown that

$$V_0(t_2-t_1) = 2h - \frac{hx^2}{4d^2-h^2}.$$

The magnitude of $\dfrac{hx^2}{4d^2-h^2}$ is small and has for our experiments the approximate value of 6·5 ft.

Hence h can be determined from the approximation

$$h = \frac{V_0(t_2-t_1)}{2} + 3,$$

where h, V_0, t_2 and t_1 are in foot-second units.

The slant range $R = \surd(h^2+x^2)$, and thence, from (1), d can be determined.

A more direct determination of the depth of the sea could be obtained from the difference in the times of arrival of the sounds arriving by paths (*a*) and (*d*) but for the fact that the arrival (*d*) did not have a sharp beginning.

In six out of the twelve records obtained with the depth charges, the time of arrival of the sound by path (*c*) could not be accurately resolved. However, from the other six records a mean sinking rate for the charges could be determined, and this provided, with increased error, the calculation of firing depth. A check on the depths of firing of the charges was obtained from preliminary tests which were carried out by the Admiralty Mining Establishment on four of the pistols from the same batch as those provided for these experiments. These tests gave a mean value of firing of 860 ft. (262 m) with a probable error of ± 20 ft. (± 6 m). The values obtained from our observations are shown in table 1.

TABLE 1

shot no.	1 depth from time observation (ft.)	2 sinking rate (ft./s)	3 depth calculated from mean sinking rate (ft.)
1	913 ± 12	9·32	—
2	(873*)	—	810 ± 50
3	—	—	854 ± 50
4	(869*)	—	935 ± 50
5	(873*)	—	890 ± 50
6	874 ± 12	8·57	—
7	866 ± 12	8·21	—
8	938 ± 12	9·29	—
9	857 ± 12	9·32	—
10	854 ± 12	8·71	—
11	—	—	908 ± 50
12	—	—	908 ± 50
mean	884 (269 m)	8·90 S.D. = 0·43 (2·71 m/s, S.D. = 0·13)	884 (269 m)

* Poor observation, rejected in favour of other determination.

The error in column 1 of this table is estimated from the accuracy with which the times could be measured. For those in column 3 it has been assumed that the error will be proportional to the standard deviation of the observations in column 2.

The correction calculated for the slant range between the charge and ship's geophone must be added to all arrivals. Unlike the other small corrections which must be applied it is independent of the bottom structure. These other corrections arise from two causes: first, the variation in the charge firing depth, and secondly, the changes in the water depth along the line. In calculating the corrections for these variations it is necessary to know the approximate angle of inclination of the ray which is producing the arrival, and this is obtained by measuring the slope of the branch of the time-distance curve on which the arrival lies. The inclination of the ray is equal to the inverse sine of the slope. In the correction for the variations of the depth of sea, assumptions must be made concerning which layer of the sea bed is causing them. Since they were small, it has been assumed that they represent alterations in the sediment thickness and that the interface below is a plane surface. It has also been assumed that the bedding planes in the surface layers are parallel to the sea bed, and not necessarily horizontal.

In the calculation of the corrections for depth of sea to be applied to the multiple refractions following the direct sound through the water it is necessary to determine the depth at each point where the ray path enters and leaves the sea bed, and then to sum the total variation from the mean depth for the complete track. This process was of sufficient importance to reduce the residuals from the calculated curves markedly. For the other arrivals the correction only involves the variations from the mean depth below the shot point and hydrophone. For the multiple refractions the values to be added to the arrival times of the direct sound and the refracted sound are respectively,

$$\Delta T_D = (\Delta h - \Sigma \Delta d)\frac{\tan\theta_1}{V_1},$$

$$\Delta S = (\Delta h - \Sigma \Delta d)\frac{\sec\theta_1}{V_1}.$$

For the other arrivals the time spent on the inclined part of the track is independent of distance, and it is only necessary to correct the refraction arrival times by

$$\Delta S = \frac{\Delta h}{V_1}\cos\theta_1 - \Sigma\Delta d\left(\frac{\sec\theta_1}{V_1} - \frac{\sec\theta_2}{V_2} + \frac{\tan\theta_2}{V_3}\right).$$

In these expressions

ΔT_D = correction to be added to the time of arrival of the direct sound,
ΔS = correction to be added to the time of arrival of the refracted sound,
Δh = variation from mean in depth of shot,
$\Sigma\Delta d$ = sum of the differences from mean depth of water for all points where the ray enters and leaves the sea bed,
θ_1 = inclination of ray on the sea bed,
V_1 = velocity of sound in the water,
θ_2 = inclination of ray at the base of the surface layer of the sea bed,
V_2 = velocity at the base of the surface layer of the sea bed, and
V_3 = velocity at the deepest point of the ray.

In these corrections the effect of the variations in the angle of the sea bed from the horizontal on the angles of inclination of the rays has been omitted. This is justifiable in our example, but would not be so if the angle of the sea bed varied from the horizontal by more than a few degrees.

In the presentation of the results, not only have the observations been corrected in the way shown above, but they have also been corrected to the values they would have had if the charge and hydrophones had been on the surface. This has been done in order that they should all be directly comparable.

4·3. *Refraction shooting station* 1, 15 *August* 1949

The estimated position of this line is shown in figure 4. This position is doubtful on account of the day being overcast and the only sights which were obtained being at the time when the buoys were being recovered, 6 h after being launched. It is possible, therefore, that the position is wrong by as much as five sea miles.

Table 2. Refraction shooting station 1, 15 August 1949

Ship's speed 12·9 ft./s; hydrophone depth 65 ft.

	shot number				
	1	2	3	4	5
(*a*) drop time (s)	98	91	96	105	100
(*b*) horizontal distance ship to charge (ft.)	1360	1270	1340	1450	1390
(*c*) time of arrival bottom reflexion (s)	2·70	2·765	2·720	2·690	2·660
(*d*) difference in time between (*c*) and time of arrival surface bottom reflexion (s)	0·371	—	—	—	—
(*e*) depth of charge (ft.)	913	810*	854*	935*	890*
(*f*) depth of water (ft.)	7870	7920	7860	7900	7760
mean depth of water (ft.)	—	—	7862	—	—

* Calculated from mean sinking rate.
Line (*b*) includes 100 ft. for distance between internal geophone and depth-charge chutes.

Uncorrected arrival times (*s*)

shot no.	buoy no.	1st arrival	2nd arrival	direct sound	order no. of multiple refraction 0	1	2	3	4	5
1	2	6·92	—	7·17	7·71	9·46	11·50*	—	—	—
	3	7·24	—	8·27	8·81	10·49	—	—	—	—
2	1	7·72*	8·09	9·21	—	10·82*	—	—	—	—
	2	7·94	9·62	10·55	10·88	—	14·35*	—	—	—
	3	8·22	—	11·70	12·09	—	13·25*	—	—	—
3	2	9·20*	—	13·56	13·90	14·96	16·36	—	—	—
	3	10·11*	12·73	14·78	15·07	16·02	17·36	19·88	—	—
4	2	—	14·51	16·86	17·11	17·98	19·44	21·37	23·96	—
	3	—	15·21	18·11	18·35	19·18	20·49	22·35*	24·81*	—
5	2	—	17·54	20·08	20·30	21·03	22·21	23·68	26·30	—
	3	—	—	21·36	21·56	22·26	23·41	25·13	26·89	29·16*

* Poor observation, probably late.

Five charges were fired, and the results obtained from them provided first arrival points up to a distance of 13 sea miles, and the multiple refractions to a range of about 20 sea miles. These results were not as complete as on the subsequent occasions on account of the buoy amplifiers being partially blocked by radio feedback; this had the effect of reducing the amplifier sensitivity, and it was eliminated on subsequent occasions.

Inexperience in handling the buoys from the *Weather Explorer* resulted in damage to three of the buoys which necessitated their recovery and, but for one which was damaged beyond immediate repair, relaunching. The line was therefore shot with three out of the four buoys.

TABLE 3. REFRACTION SHOOTING STATION 1, 15 AUGUST 1949

(Arrival times in seconds; fully corrected for hydrophones and charges on the surface, and for mean depth of water, 7862 ft.)

shot no. ...	1	1	2	2	2	3	3	4	4	5	5
buoy no. ...	2	3	1	2	3	2	3	2	3	2	3
1st arrival ...	7·42	7·74	8·22*	8·44	8·72	9·71*	10·62*	—	—	—	—
2nd arrival ...	—	—	8·58	10·04	—	—	13·16	14·97	15·67	17·98	—
direct sound ...	7·50	8·60	9·52	10·86	12·01	13·88	15·10	17·21	18·46	20·41	21·69
equivalent distance ... (ft.)	36900	42300	46800	53400	59100	68300	74300	84700	90800	100400	106700
Order 0											
direct sound	8·10	9·25	—	11·52	12·75	14·66	15·88	18·28	19·53	21·44	23·14
arrival	8·67	9·83	—	11·97	13·16	15·02	16·19	18·55	19·79	21·68	23·36
Order 1											
direct sound	7·75	8·88	—	—	—	14·28	15·54	17·84	19·07	20·99	22·27
arrival	10·11	11·16	—	—	—	15·72	16·89	19·00	20·18	21·98	23·21
Order 2											
direct sound	—	—	—	—	—	14·26	15·51	17·68	18·93	20·86	22·14
arrival	—	—	—	—	—	17·11	18·14	20·30	21·35	23·03	24·23
Order 3											
direct sound	—	—	—	—	—	—	15·30	17·42	18·73	20·71	21·99
arrival	—	—	—	—	—	—	20·49	22·03	23·04*	24·37	25·82
Order 4											
direct sound	—	—	—	—	—	—	—	17·39	18·64	20·58	21·86
arrival	—	—	—	—	—	—	—	24·58	25·43	26·89	27·48
Order 5											
direct sound	—	—	—	—	—	—	—	—	—	—	—
arrival	—	—	—	—	—	—	—	—	—	—	—

* Poor observation, probably late.

The fifth-order multiple refractions have been omitted from this table since there were not enough points to allow the measurement of the slope of the curve on which they lay; for this reason the corrections could not be calculated.

Table 2 shows the basic information obtained from the shots. Table 3 gives the arrival times corrected, as shown in the preceding paragraph, so as to be equivalent to the results which would be obtained if the charges and hydrophones were at the surface. For some of the observations on this day there was doubt as to the exact instant when a new phase started, and in these cases the observed time will be later rather than earlier than the true time of arrival. These observations are asterisked in the tables. Figure 6 shows the corrected arrivals plotted against the time of arrival of the direct sound through the water; these times are directly proportional to the distance between the shots and buoys.

4·4. *Refraction shooting station* 2, 18 *August* 1949

The position of this line is shown in figure 4. It was intended that it should be in the same place as the line of station 1, but the sights which were obtained during the experiments showed it to be displaced southwards.

The radio to amplifier feedback which caused trouble at station 1 was on this occasion eliminated, and as a result the effective sensitivity was considerably higher. Four buoys were launched, and five depth charges were fired. First arrival points were recorded up to a maximum range of about 20 sea miles. The depth of water under the buoys was not obtained, and it was necessary to extrapolate from neighbouring soundings, and from the measured depths at the shot points. The hydrophone depth was increased to 130 ft. for the experiments on this day.

TABLE 4. REFRACTION SHOOTING STATION 2, 18 AUGUST 1949

Ship's speed 14·2 ft./s; hydrophone depth 130 ft.

	shot number 1	2	3	4	5
(*a*) drop time (s)	102	105½	101	92	98
(*b*) horizontal distance ship to charge (ft.)	1550	1600	1540	1410	1490
(*c*) time of arrival bottom reflexion (s)	2·75	2·73	2·68	2·65	2·70
(*d*) difference in time between (*c*) and time of arrival surface bottom reflexion (s)	0·355	0·352	0·381	0·348	0·347
(*e*) depth of charge (ft.)	874	866	938	857	854
(*f*) depth of water (ft.)	8035	7995	7900	7715	7870
mean depth of water (ft.)	—	—	7902	—	—
extrapolated depth under buoys (ft.)	—	—	8030	—	—

Line (*b*) includes 100 ft. for distance between internal geophone and depth-charge chutes.

Uncorrected arrival times (*s*)

shot no.	buoy no.	1st arrival	2nd arrival	3rd arrival	direct sound	shot number 0	1	2	3	4	5	6
1	1	< 6·01	—	—	6·01	—	8·92	—	—	—	—	—
	2	6·53	—	—	6·65	—	9·33	—	—	—	—	—
	3	6·94*	—	—	7·26	7·86	9·76	—	—	—	—	—
	4	7·25	—	—	8·53	9·03	—	—	—	—	—	—
2	1	7·59	—	—	9·66	10·13	11·58	13·90	—	—	—	—
	2	7·80	—	—	10·34	10·79	12·28	14·41	—	—	—	—
	3	7·96	—	—	10·85	11·27	12·54	14·89	—	—	—	—
	4	8·33	—	—	12·15	12·55	13·72	15·59	—	—	—	—
3	1	8·54	11·79	—	12·97	13·32	14·43	16·14	18·77	—	—	—
	2	8·75	11·96	—	13·66	13·93	15·02	16·78	19·23	—	—	—
	3	8·95*	—	—	14·12	14·44	15·46	17·24	19·44	—	—	—
	4	9·14	13·88	—	15·44	—	—	—	—	—	—	—
4	1	—	—	—	16·36	16·64	17·54	19·11	20·79	23·71	—	—
	2	9·50	12·54	14·97	17·04	17·31	18·20	19·68	21·56	24·16	—	—
	3	—	—	—	17·52	17·78	18·60	20·05	21·92	24·45	—	—
5	2	10·37	14·08	{16·20, 17·90	20·63	20·81	21·54	22·76	24·53	26·51	29·28	—
5	3	—	16·65	18·60	22·41	22·61	23·27	24·37	25·94	27·87	30·21	32·69

* Poor observation, probably late.

Table 4 shows the uncorrected and basic information obtained from these shots, and table 5 gives the arrival times, corrected, as before, to the values which would have been obtained if the hydrophones and charges had been at the surface, and for the variations in water depth along the line. These last variations were larger than on the first day, and showed a total scatter of 300 ft. The observations are plotted in figure 7.

4·5. *Refraction shooting station* 3, 22 *August* 1949

For this day's work only two depth charges remained, and it was decided that they should be used at the larger ranges to confirm the results obtained at the longer ranges at station 2.

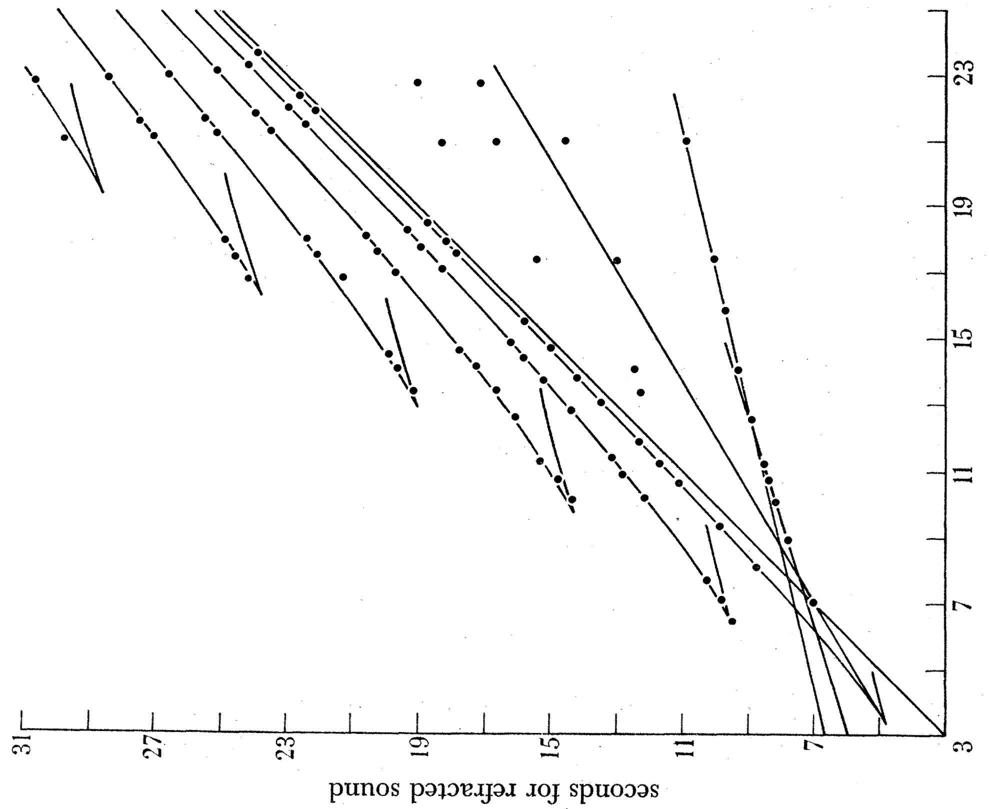

FIGURE 7. Station 2: experimental observations and calculated curves.

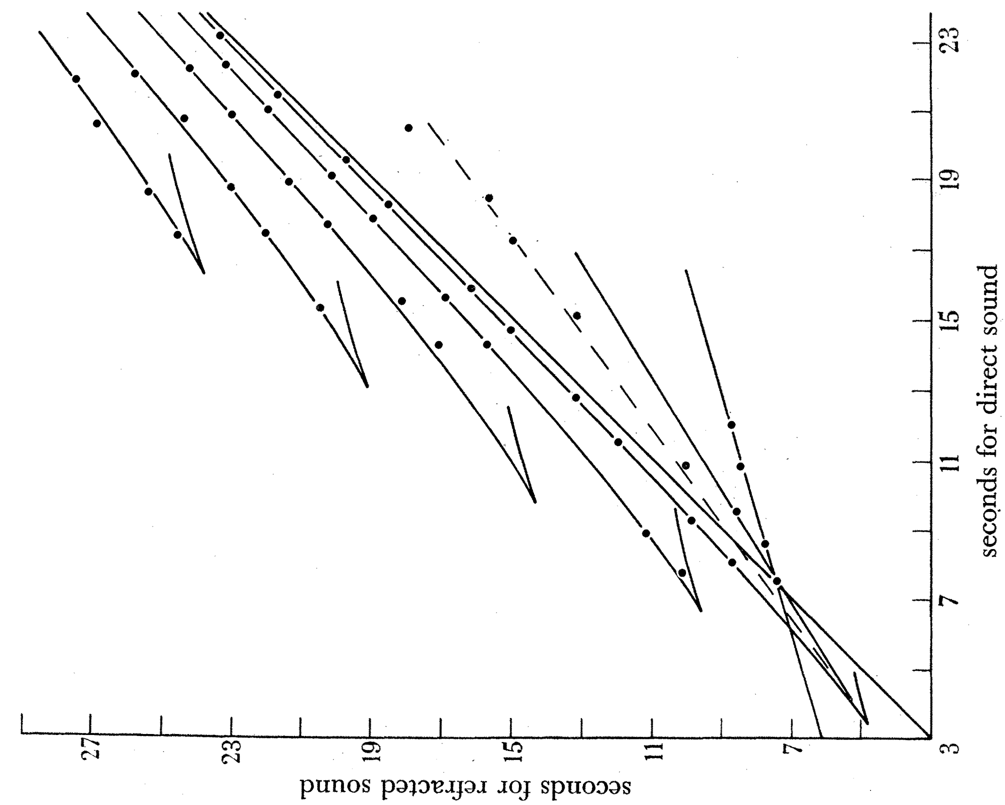

FIGURE 6. Station 1: experimental observations and calculated curves.

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TABLE 5. REFRACTION SHOOTING STATION 2, 18 AUGUST 1949

(Arrival times in seconds; fully corrected for hydrophone and charges on the surface, and for mean depth of water 7900 ft.)

shot no. ...	1	1	1	1	2	2	2	2	3	3	3	3	4	4	4	5	5	5
buoy no. ...	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	2	3	4
1st arrival ...	—	7·01	7·47*	7·78	8·14	8·35	8·51	8·88	9·07*	9·28	9·48*	9·67	—	10·04	—	10·91	—	—
2nd arrival ...	—	—	—	—	—	—	—	—	12·26	12·43	—	14·35	—	13·03	—	14·58	—	17·15
3rd arrival ...	—	—	—	—	—	—	—	—	—	—	—	—	—	15·43	—	16·70} 18·37}	—	19·07
direct sound ...	6·37	7·01	7·62	8·89	10·03	10·71	11·22	12·52	13·33	14·02	14·48	15·80	16·69	17·37	17·85	20·98	21·47	22·76
equivalent dist. (ft.) ...	31300	34500	37500	43700	49300	52700	55200	61600	65600	69000	71200	77700	82100	85500	87800	103200	105600	112000
Order 0																		
direct sound	—	—	8·10	9·30	10·65	11·21	11·83	13·02	13·78	14·68	15·14	—	17·59	17·89	18·49	21·91	22·40	23·67
arrival	—	—	8·73	9·83	11·15	11·69	12·28	13·45	14·17	14·97	15·48	—	17·88	18·19	18·78	22·12	22·62	23·90
Order 1																		
direct sound	6·44	7·08	7·71	—	10·18	10·91	11·40	12·76	13·69	14·37	14·83	—	17·11	17·79	18·26	21·52	22·00	23·27
arrival	9·41	9·82	10·26	—	12·14	12·85	13·12	14·36	15·17	15·75	16·19	—	18·32	18·98	19·36	22·47	22·96	24·16
Order 2																		
direct sound	—	—	—	—	10·09	10·78	11·28	12·59	13·44	14·14	14·62	—	16·94	17·62	18·08	21·32	21·81	23·10
arrival	—	—	—	—	14·39	14·78	15·35	16·06	16·65	17·29	17·77	—	19·72	20·29	20·65	23·51	23·95	25·12
Order 3																		
direct sound	—	—	—	—	—	—	—	—	13·36	14·06	14·52	—	16·83	17·52	17·99	21·23	21·72	23·01
arrival	—	—	—	—	—	—	—	—	19·19	19·65	19·87	—	21·30	22·08	22·42	25·18	25·52	26·58
Order 4																		
direct sound	—	—	—	—	—	—	—	—	—	—	—	—	16·78	17·45	17·94	21·13	21·62	22·91
arrival	—	—	—	—	—	—	—	—	—	—	—	—	24·17	24·61	24·89	27·06	27·48	28·41
Order 5																		
direct sound	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	21·07	—	22·84
arrival	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	29·79	—	30·69

* Poor observation, probably late.

TABLE 6. REFRACTION SHOOTING STATION 3, 22 AUGUST 1949

Ship's speed 14·8 ft./s; hydrophone depth 130 ft.

(*a*) drop time (s)	102	102
(*b*) horizontal distance, ship to charge (ft.)	1607	1607
(*c*) time of arrival bottom reflexion (s)	2·645	2·635
(*d*) difference in time between (*c*) and time of arrival of surface bottom reflexion	—	—
(*e*) depth of charge (ft.)	908	908
(*f*) depth of water (ft.)	7825	7805
mean depth of water (ft.)	7815	
mean depth of water under buoys (ft.)	7940	

Line (*b*) includes 100 ft. for distance between internal geophone and depth-charge chutes.

Since the arrivals in line (*d*) could not be accurately resolved the depth of firing is obtained from the mean sinking rate.

Uncorrected arrival times (*s*)

shot no.	buoy no.	1st arrival	2nd arrival	direct sound	multiple refractions 0	1	2	3	4	5	6
1	1	7·66	10·18	12·16	12·52	13·65	15·35	18·02	—	—	—
	2	8·22	11·40	13·90	14·25	15·25	16·90	19·15	—	—	—
	3	8·62	12·65	15·59	15·87	16·78	18·23	20·33	22·98	—	—
2	1	9·71	15·71	20·09	20·32	21·04	22·21	23·94	26·06	28·71	—
	3	11·06*	—	23·54	23·72	24·36	25·38	26·84	28·72	30·97	33·67

* Poor observation, probably late.

TABLE 7. REFRACTION SHOOTING STATION 3, 22 AUGUST 1949

(Arrival times in seconds; fully corrected for hydrophone and charges on the surface, and for mean depth of water, 7·820 ft.)

shot no. ...	1	1	1	2	2
buoy no. ...	1	2	3	1	3
1st arrival ...	8·23	8·79	9·19	10·28	11·63*
2nd arrival ...	10·70	11·92	13·15	16·23	—
direct sound ...	12·53	14·27	15·96	20·46	23·91
equivalent dist. (ft.) ...	61600	70200	78500	100700	117600
Order 0					
direct sound	14·20	15·94	16·97	21·31	24·76
arrival	14·57	16·30	17·26	21·56	24·96
Order 1					
direct sound	12·92	14·70	16·45	20·94	24·39
arrival	14·46	16·07	17·68	21·93	25·25
Order 2					
direct sound	12·78	14·52	16·27	20·84	24·29
arrival	16·05	17·58	18·96	23·01	26·18
Order 3					
direct sound	12·72	14·46	16·15	20·74	24·19
arrival	18·66	19·79	20·97	24·65	27·55
Order 4					
direct sound	—	—	16·13	20·62	24·14
arrival	—	—	23·61	26·68	29·38
Order 5					
direct sound	—	—	—	20·62	24·07
arrival	—	—	—	29·33	31·59
Order 6					
direct sound	—	—	—	—	—
arrival	—	—	—	—	—

* Poor observation, probably late.

The position of the line was altered to an area to the north and east of that of the previous lines. First arrival points were obtained between 12 and 20 sea miles.

The first buoy to be laid ceased functioning before the first shot was fired. This shot was therefore fired with only three buoys in operation. For the second shot, recordings were obtained from two buoys. The failure to record the signal from the third buoy resulted indirectly from a break developing in the feeder cable from the receiving aerials in the ship. This break occurred after the charge had been dropped and before it fired. A repair was effected in the one minute which was available, but as a result of this hurried repair a jack plug was inadvertently partially removed from its socket in the recording camera and the output of one of the receivers was thereby disconnected.

The basic information obtained from these shots is shown in table 6, and the corrected results in table 7. Figure 8 shows the corrected observations plotted against the time of arrival of the direct sound.

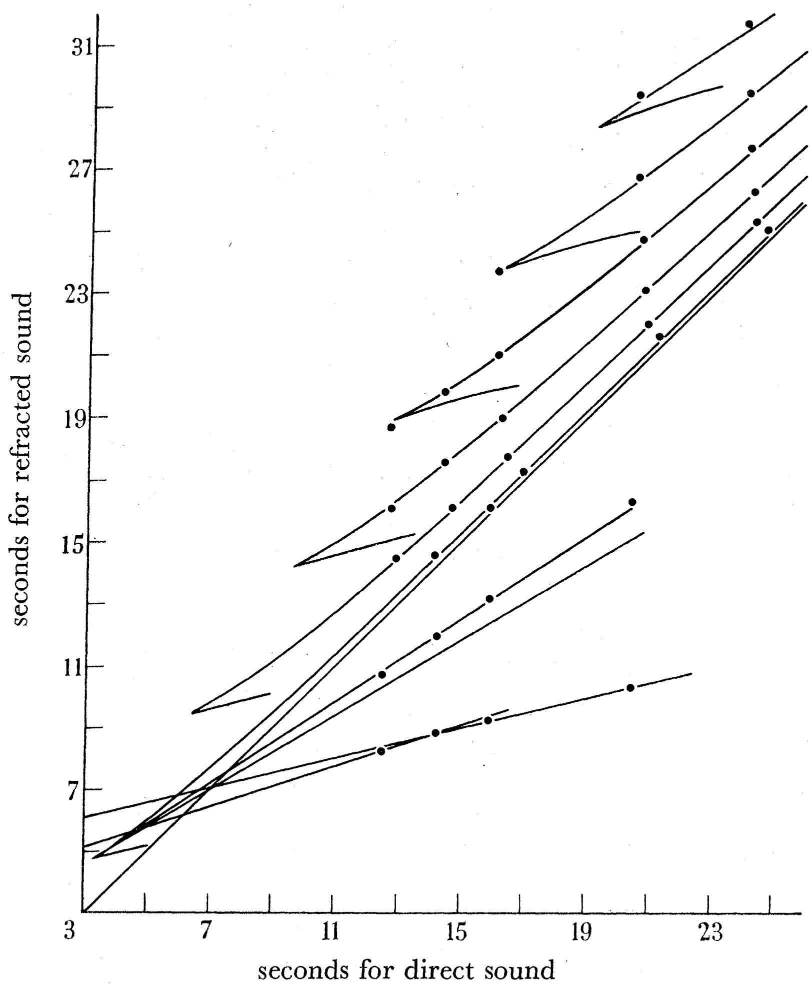

FIGURE 8. Station 3: experimental observations and calculated curves.

5. THE REDUCTION OF THE OBSERVATIONS

The experimental results shown in the last section fall into two separate groups which involve different arithmetical treatment. These two groups are

(*a*) The phases which follow the direct sound through the water.

(*b*) The phases which precede the direct sound through the water.

Since the phases in group (*a*) belong to the topmost layer of the sea bed they will be considered first.

5·1. *The arrivals following the direct sound through the water*

The multiple pulses following the direct sound through the water were, when the records were first inspected, assumed to represent sound paths which were reflected from the surface and sea bed. On this assumption the first arrival after the direct sound would have been reflected once from the bottom, the second pulse twice from the bottom and once from the surface, and the nth pulse n times from the bottom and $(n-1)$ times from the surface. A closer analysis of the times of arrival of these multiple arrivals showed two features which indicated that this simple explanation was not correct. First, the times of arrival were significantly different from those expected on this hypothesis and secondly, the number of pulses for any one distance between shot point and receiver was limited. On the multiple-reflexion hypothesis this number would be unlimited unless, when the angle of inclination of the sound ray on the bottom became less than the critical angle, total transmission took place. Unsuccessful attempts were made to explain the limited number of multiple pulses on this basis, but the hypothesis was not tenable. It was further shown that the possible velocity gradients in the water were inadequate to explain the differences.

The time differences from the calculated curves for the simple reflexion hypothesis might have been explained by postulating the existence of a layer providing strong reflexions below the sea bed which itself had a small reflexion coefficient. This hypothesis, while providing for a particular depth of reflecting horizon an improvement in the fit of the calculated curves to the observations, again resulted in systematic residuals of greater magnitude than the errors of observation. Other multiple-reflexion hypotheses were tried, such as inclining the horizon of strong reflexions, but in each the residuals remained systematic and significant.

Figure 9 shows the observations of the multiple refractions for station 2, together with the calculated curves for the bottom to surface reflexion hypothesis and for the case of a reflecting interface 500 ft. (150 m) below the bottom. For this last case it has been assumed that the velocity of sound in the sea bed above the interface is the same as that of sound in the sea. From this figure, it is apparent that the curves marked (*a*) do not provide a good fit at the smaller ranges, while those marked (*b*) show a small systematic divergence from the observations at the greater ranges.

With the rejection of the simple reflexion hypotheses the next postulate was that there was a linear gradient of velocity in the sea bed, and that the ray paths were bent into circular tracks. Tracks of this type have been drawn in figure 10.

The calculated curves showed immediately that a good fit could be obtained by selecting an appropriate value for the gradient of velocity, and assuming that the velocity in the sea-bed surface had the same value as that of the water. With this hypothesis it was theoretically impossible to have more than a limited number of arrivals at a given distance between shot point and hydrophone, since for each curve of multiple refraction travel against time the distance (or time of arrival of the direct sound) reaches a minimum value which is greater than zero. This minimum value is at the cusp of the double-branched curve. This feature was conspicuous with the experimental results. For the velocity gradient of 2·5/s the observations and calculated curves are shown in figures 6, 7 and 8.

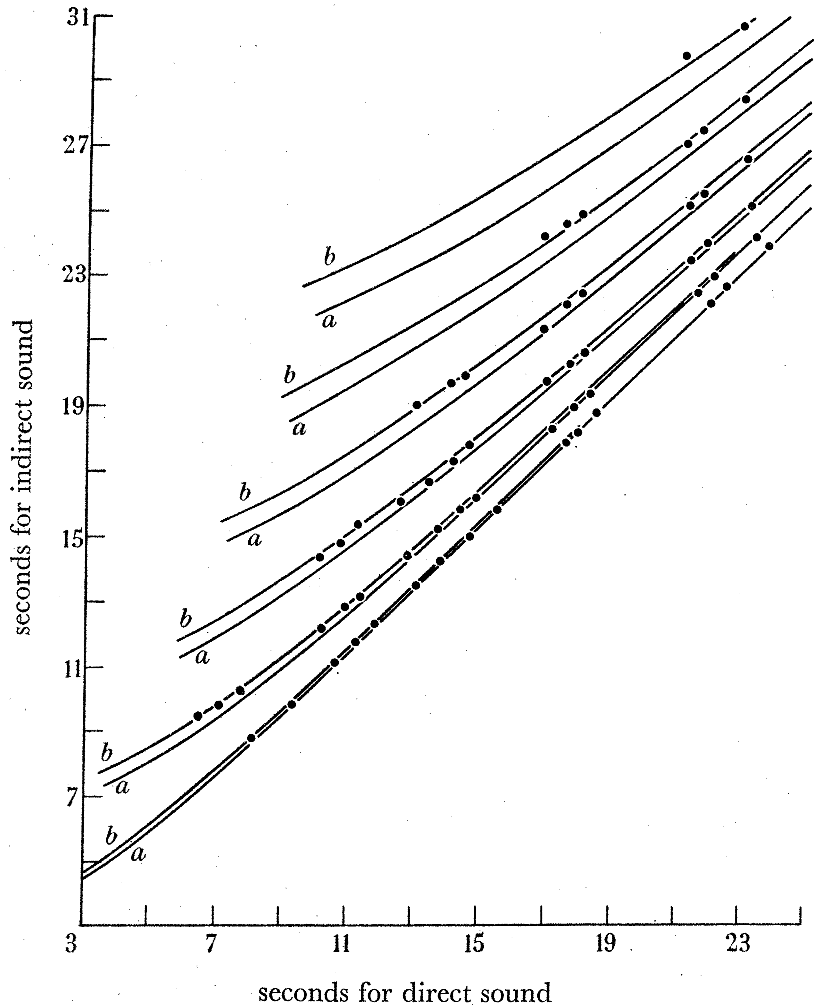

FIGURE 9. Experimental observations and calculated curves for simple reflexion from: (*a*) sea bed, (*b*) 500 ft. (150 m) below sea bed.

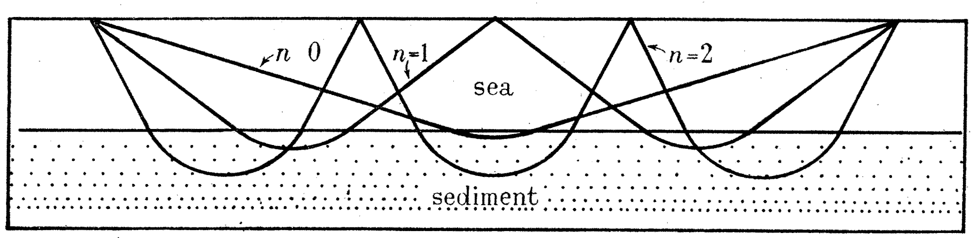

FIGURE 10. Multiple refraction paths for orders 0, 1 and 2.

Further support for the hypothesis that these phases represent refractions rather than reflexions lies in their amplitude. The solid angle subtended at the source by a receiver of finite size must be smaller for the reflected paths than for the direct sound, while for the curved paths produced by a velocity gradient in the layer below the sea bed the angle can be greater than that of the direct sound. Provided therefore that the attenuation in this layer is low, the refracted pulses can be larger in amplitude than the direct sound; this was a marked feature of our records. For a velocity in the layer below the sea bed which increases linearly with depth from a value equal to that of sound in water,

$$D = 2h_1 \tan \theta_1 + \frac{2V_1}{a} \cot \theta_1$$

$$S = \frac{2h_1}{V_1} \sec \theta_1 + \frac{2}{a} \sinh^{-1} \cot \theta_1,$$

where D = horizontal distance between shot point and receiver,
S = time between instant of explosion and arrival at the receiver of the refracted waves,
h_1 = depth of sea,
θ_1 = angle of inclination of the ray on the sea bed,
V_1 = velocity of sound in the sea, and
a = the gradient of velocity in the layer below the sea.

Considering the two-dimensional problem of a vertical plane passing through the source and receiver and assuming the receiver to be circular and of diameter small compared with the distance, it can be shown that the angle subtended for the curved ray will always be greater than that for the direct sound provided

$$\frac{V_1}{ah_1} > \frac{1}{(1 + \operatorname{cosec} \theta_1)^2}.$$

This will always be true if $V_1/ah_1 > \frac{1}{4}$.

In our example the value of V_1/ah_1 is approximately $\frac{1}{4}$, and therefore except for the values of θ_1 approaching $\frac{1}{2}\pi$ the angle subtended along the curved ray will be greater than that for the direct sound. The angle is greatest for a particular value of θ_1 given by the relationship $\tan^2 \theta_1 = V_1/ah_1$.

With this angle both $\partial D/\partial \theta_1$ and $\partial S/\partial \theta_1$ are zero. This means that not only is there simple focusing, but also that the energy arriving by this path will not be spread out in time. If, therefore, preceding phases do not obscure the arrivals they will have a sharp beginning.

In figures 11, 12 and 13 the observations of the multiple refractions are shown reduced to zero order with the time of arrival of the direct sound through the water subtracted from them. Thus if

T_D is the time of arrival of the direct sound,
S is the time of arrival of the nth-order refraction,

the abscissae of these figures represent the time $\dfrac{S - T_D}{(n+1)}$,

and the ordinates the time $\dfrac{T_D}{(n+1)}$.

With this method the observations are all placed on one curve, and the differences between the curves calculated for various gradient values made more conspicuous. In these figures the gradient values of the calculated curves are 1·0, 2·5 and 4·0/s.

The observations for station 1 show a considerable scatter about the calculated curve which was caused by the low sensitivity of the buoy amplifiers at this station. Nevertheless, it can be seen that the best value for the gradient, assuming it to be constant, is about 2·5/s. The observations for the other two stations (figures 12 and 13) have considerably less scatter, and it has been found that with a gradient value 2·5/s the fit is such that

(*a*) The introduction of a term in the velocity-depth relationship containing the square of the depth would not provide a significant improvement.

(*b*) The value of the gradient has an estimated error $\pm 0{\cdot}2$/s, provided a linear relationship between velocity and depth is assumed.

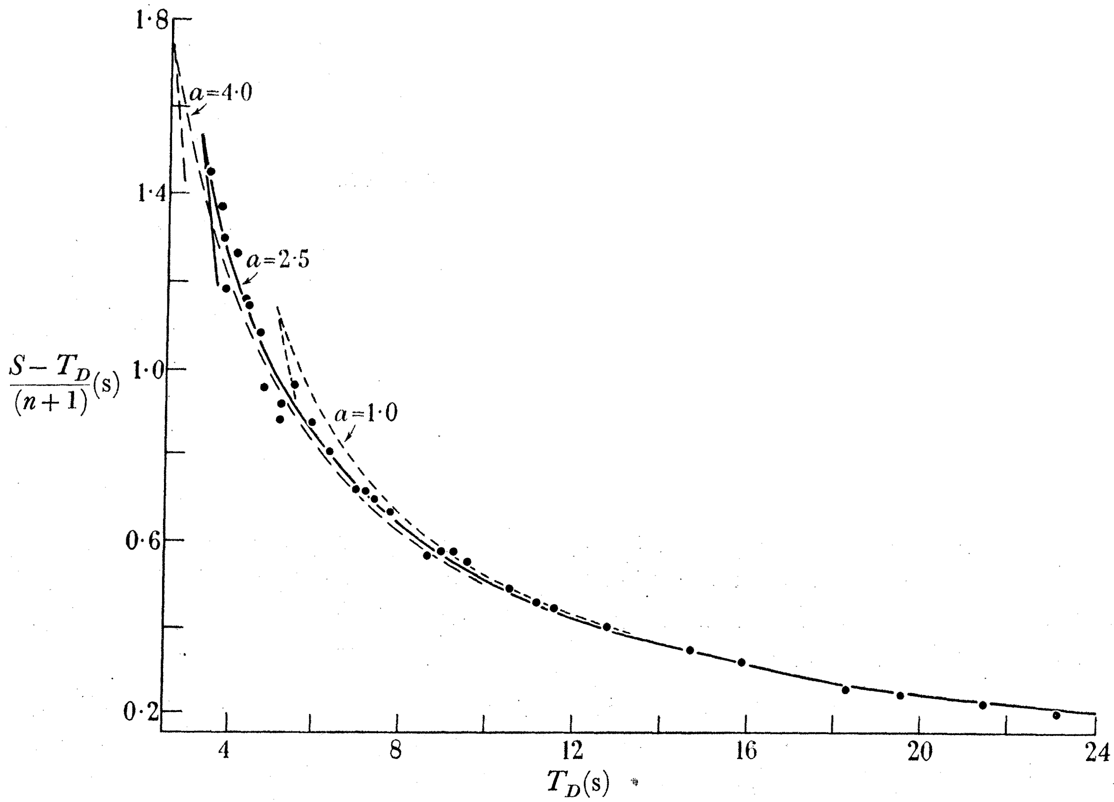

FIGURE 11. Station 1: experimental results and curves calculated for gradient values 1·0, 2·5 and 4·0/s.

In selecting the best value for the velocity gradient, there are two considerations of primary importance. First, the gradient must be such that the cusp of the time-distance curve calculated for this gradient must lie at a distance less than the minimum observed distances for the multiple refractions after their reduction to zero order. Secondly, since the more distant points are of less weight than the closer, the value of the gradient should be such that the fit is better for the closer points. The weights of the points are different, since the larger-range observations have all been obtained from the lower-order refractions, with the result that any fixed errors have, at these larger ranges, been divided by the smaller integers.

The justification for assuming that there is no change in velocity at the interface between the water and the sediments lies in the fact that if there were a change then the calculated

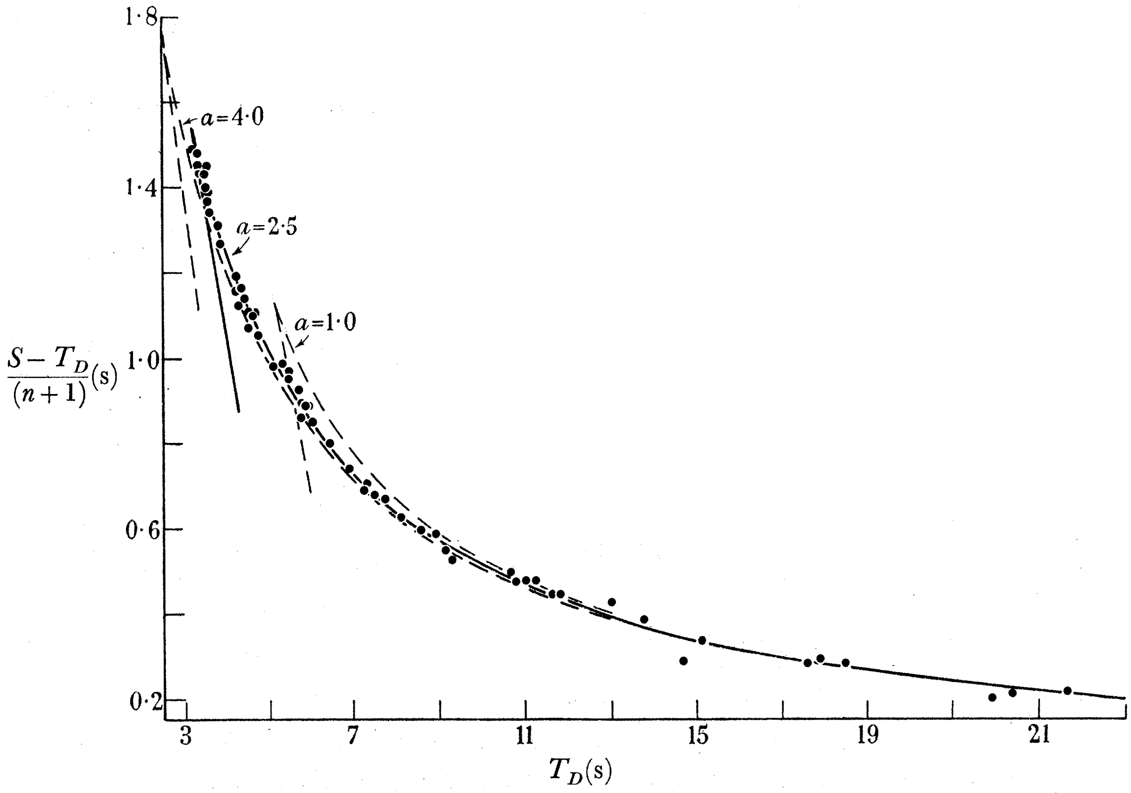

FIGURE 12. Station 2: experimental results and curves calculated for gradient values 1·0, 2·5 and 4·0/s.

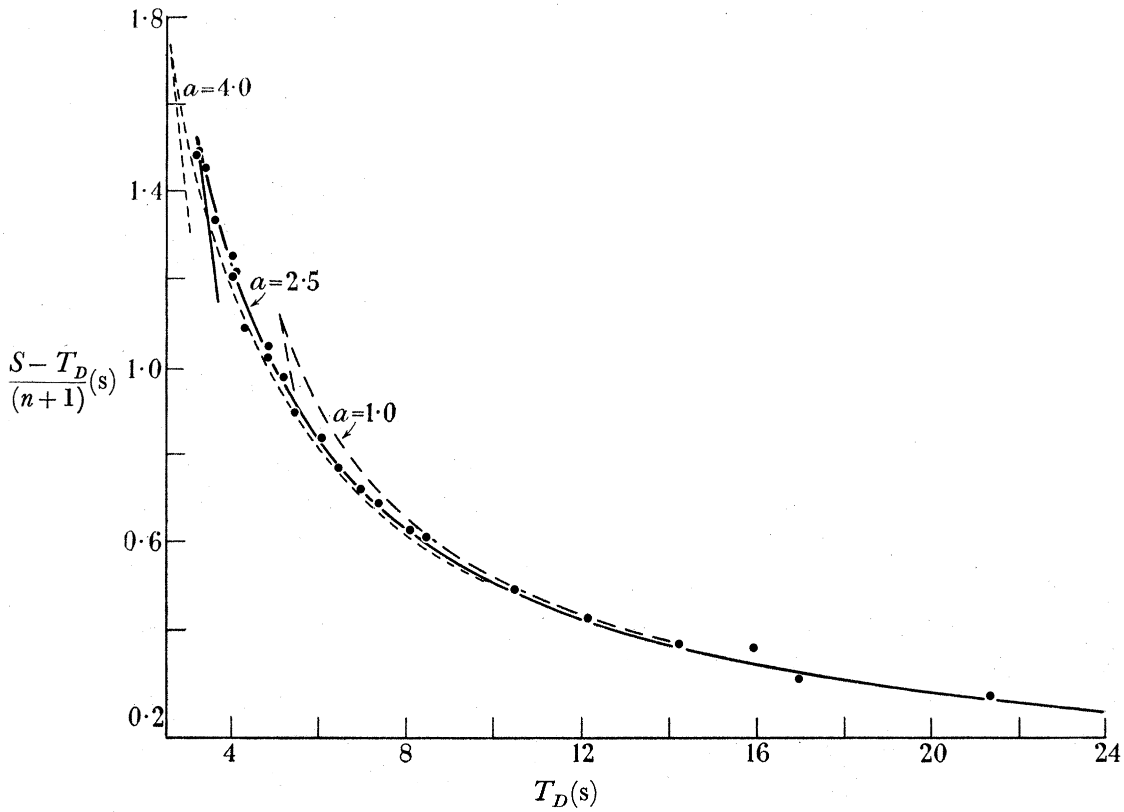

FIGURE 13. Station 3: experimental results and curves calculated for gradient values 1·0, 2·5 and 4·0/s.

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curves of S against T_D would not, at the larger values of T_D, become asymptotic to a line of unit slope passing through the origin.

The equations relating the times for the direct sound and the refracted sound are, respectively,

$$T_D = \frac{2h_1}{V_1}\tan\theta_1 + \frac{2V_2(0)}{aV_1}\cot\theta_2(0),$$

$$S = \frac{2h_1}{V_1}\sec\theta_1 + \frac{2}{a}\sinh^{-1}(\cot\theta_2(0)),$$

where $V_2(0)$ = velocity of sound in the surface of the sea bed,

$\theta_2(0)$ = angle of inclination of the ray on the lower side of the water to sea bed interface.

These expressions can be written as

$$T_D = \frac{2h_1}{V_1}\tan\theta_1 + \frac{2}{a}\sqrt{\{\cot^2\theta_1 + (1-m^2)\}},$$

$$S = \frac{2h_1}{V_1}\sec\theta_1 + \frac{2}{a}\sinh^{-1}\left[\frac{\sqrt{\{\cot^2\theta_1 + (1-m^2)\}}}{m}\right],$$

where $V_2(0) = mV_1$.

For the case where $m > 1$ the asymptote of the curve relating T_D and S is the line of slope $1/m$, and the calculated curve will never have a slope greater than this value. The slope of the curve on which the observations lie, however, has a minimum slope $> 0{\cdot}97$. This means that if there is an increase in velocity on passing through the interface, its magnitude cannot be greater than $0{\cdot}03V_W$. For the case where $m < 1$ the asymptote is a line of unity slope which does not pass through the origin. The displacement of the asymptote is such that a decrease in velocity on passing through the interface greater than $0{\cdot}05V_W$ would be conspicuous.

The fact that the scatter is large and number of observations small at the greater values of T_D results in uncertainty in the gradient of velocity at the shallower depths of penetration of the rays. This uncertainty is further increased by the fact that $\partial S/\partial a$ becomes smaller as θ_1 increases, and therefore as T_D increases. This results in the calculated curves for differing gradient values converging on one another as the range increases.

It is, however, apparent from figure 14, in which the depth of penetration for station 2 is plotted against T_D for a gradient of 2·5/s, that the uncertainty is confined to a depth not greater than 100 ft. Further, changes in velocity within this thickness would be conspicuous (except if there were a continuously changing gradient which changed sign within this thickness) in the same way that sudden changes on passing through the water to sea-bed interface would be apparent.

There is no evidence from the observations that any of the multiple refractions provide points on the lower limbs of the calculated curves. If there were such points they should be resolved, since they would arrive in advance of the points on the upper limb. This indicates that the velocity gradient does not continue indefinitely at a constant value but becomes zero (or possibly negative) at a particular depth. The value of the velocity (and depth) at which the gradient changes value can be determined by measuring the limiting value of the slope of the curve of $(S - T_D)$ against T_D (figures 11, 12 and 13) at the point where $(S-T_D)$ is maximum. This measurement is not precise, and if this were the only method of determining the value then it would have an estimated error of $\pm 10\%$. Consideration, however, of the arrivals occurring before the direct sound provide much finer limits, as will be shown in the next paragraph.

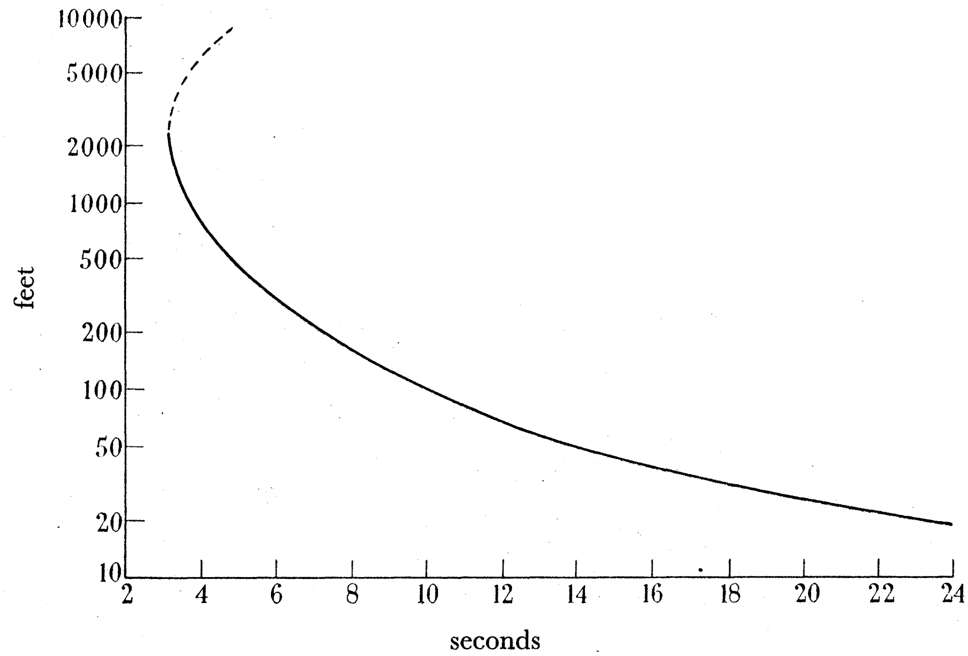

FIGURE 14. Depth of penetration of refracted wave against time of arrival of direct sound for station 2 and gradient 2·5/s.

The deduction that the velocity change at the interface is small indicates that the material on the lower side of the interface is unconsolidated and possesses a high porosity. Assuming that the value of the porosity lies between 50 and 75 % (Kuenen 1950), and that the constituent mineral particles have a density of 2·7 g/cm³, the mean density on the lower side of the interface will lie between approximately 1·8 and 1·4 g/cm³. The amplitude reflexion coefficient for equal velocities will be independent of angle of incidence and is given by the relationship

$$R=\frac{\rho_2/\rho_1-1}{\rho_2/\rho_1+1},$$

where ρ_1 and ρ_2 are the densities of the water and sea bed respectively. The value of R will therefore lie between the approximate limits 0·29 and 0·17. These limits are such that the multiple reflexions would rapidly decrease in amplitude with increasing order number and would account for their not being visible on the records in spite of the fact that they should arrive before the multiple refractions. The zero-order reflexions would be the most conspicuous, but, as figure 9 shows, could not be resolved from the zero-order refractions. Further evidence of a low reflexion coefficient was obtained from the reflexion shots in which the records were characterized by the initial amplitude of the bottom reflexion being smaller than the amplitude of the sound scattered back from below the bottom.

With other experimental results obtained in the deep sea (Ewing 1950*a*) multiple reflexions up to at least the third order have been identified, while in the same experiments

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no evidence for multiple refractions was obtained. These two facts indicate that in the western Atlantic where these experiments were carried out the consolidation with depth is less gradual than in our area.

5·2. *The phases arriving before the direct sound*

The phases which arrive before the direct sound represent paths passing along the interface beolw the sediments except for one point which is of great importance in confirming the sedimentary structure obtained from the multiple refractions discussed above. This point, which was obtained from the closest shot of station 2, represents an arrival passing along the layer of constant velocity which it was concluded must exist below the region where there was a linear increase of velocity with depth. It was fortuitous that a shot was fired exactly at the distance required to obtain this point, since if it had been half a mile in either direction no first arrival would have been obtained. It was also fortuitous that the water depth was small enough to permit this line on the time-distance curve to pass below the intersection of the lines representing the direct sound and the higher velocity layers lying below the sediments. If from this point a line is drawn through the points near the cusp of the time-distance curve of the multiple refractions (figure 7) its slope is such that, as would be required theoretically, it is tangential to the calculated curves. The position of this line is also such that it passes above the first arrival points obtained from the shots at the greater distances. If our interpretation that the velocity gradient ceased at a velocity equivalent to that near the cusp of the curve were wrong, and, in fact, the velocity increased to a higher value before reaching its limit, then it would be necessary to draw a line of smaller slope passing through a point near the cusp. This line would then pass below the first arrivals obtained at the greater ranges which would not be allowable. It would also not be possible, on this basis, to explain the single point obtained at the shortest range. We have, therefore, from the first arrivals good confirmation of the result obtained solely from consideration of the multiple refractions.

The slope of the line passing through this point and which is tangential to the multiple-refraction curve has the value of $0{\cdot}60 \pm 0{\cdot}01$, the error being estimated from the error in the observation of the first arrival and also from the error which arises in determining the best tangent through this point to the curve on which the multiple refractions lie. The point on the curve calculated for a constant velocity gradient of 2·5/s, where the slope is 0·60, lies at a greater range than the cusp of the curve and on its upper limb. Between this point and the cusp there are five observed points which should not be there if our interpretation were exact. If, however, at a shallower depth than that equivalent to this point it was assumed that the velocity gradient started to decrease in value, then it is possible to produce a calculated curve which would not increase its slope so rapidly with decreasing distance and which could therefore have the required slope at a shorter range. The existence of these five points is consequently an indication of a continuous rather than discontinuous change in velocity gradient. No attempts have been made to calculate a suitable time-distance curve of this type on account of the arithmetical complexity and also on account of the fact that two extra parameters would be introduced, the first to provide a law for the rate of change of gradient and a second to provide a point from which this rate of change should begin.

The arrivals for the three stations are shown in figures 6, 7 and 8. Table 8 gives the observations, the slopes of the lines on which they lie, the residuals, and the intercepts of the lines on the time axis. As before, these observations have been corrected for variations in depth and so that they represent the values which would have been obtained if the hydrophones and charges had been on the sea surface. The best straight lines have been calculated by the method of least squares, omitting the points for which there was uncertainty.

TABLE 8

station no.	time of arrival direct sound (s)	equivalent distance (ft.)	time of arrival refraction (s)	residuals (s)	velocity (ft./s)	intercept (s%)
1	7·50	36900	7·42	0·00	16820 ± 300	5·230 ± 0·066
	8·60	42300	7·74	−0·01	(5·14 ± 0·1	
	10·86	53400	8·44	+0·03	km/s)	
	12·01	59100	8·72	−0·01		
2	7·01	34500	7·01	—	8120 ± 150 (2·58 ± 0·05 km/s)	—
	8·89	43700	7·78	−0·01	16200 ± 200	5·090 ± 0·046
	10·03	49300	8·14	0·00	(4·94 ± 0·06	
	10·71	52700	8·35	+0·01	km/s)	
	11·22	55200	8·51	+0·01		
	12·52	61600	8·88	−0·01		
	14·02	69000	9·28	+0·02	20930 ± 400	5·967 ± 0·075
	15·80	77700	9·67	−0·01	(6·37 ± 0·12	
	17·37	85500	10·04	−0·01	km/s)	
	20·98	103200	10·91	+0·01		
3	12·53	61600	8·23	—	16200 (assumed) (4·94 ± 0·06 km/s) km/s)	4·425
	14·27	70200	8·79	+0·003	20400 ± 100	5·348 ± 0·02
	15·96	78500	9·19	−0·004	(6·22 ± 0·03	
	20·46	100700	10·28	+0·002	km/s)	

None of the poor observations have been included in this table.

For station 1, only four good first arrival points were obtained. These points gave a velocity of 16820 ± 300 ft./s (5·14 ± 0·1 km/s) and an intercept of 5·23 ± 0·07 s.

For station 2, five good points on a line giving a velocity of 16200 ± 200 ft./s (4·94 ± 0·06 km/s) were obtained at the shorter ranges. This line had an intercept on the time axis of 5·09 ± 0·05 s.

The residuals from the best straight line passing through these points were no larger than the inaccuracies in reading the times from the records, and there is, therefore, no reason to associate the standard errors of slope and intercept with variations in structure such as would be obtained if the strata were inclined to the horizontal in the direction in which the line was shot.

At distances greater than those providing the arrivals on the 16200 ft./s (4·94 km/s) line there were four further points lying on a line of slope equivalent to 20930 ± 400 ft./s (6·37 ± 0·12 km/s). The intercept of this line on the time axis was 5·97 ± 0·80 s. Again there was no evidence for structural variations causing the residuals.

For the results to be obtained from the first arrivals from station 3, only four points were available. The three most distant of these lie on a line of slope equivalent to a velocity of

20400 ± 100 ft./s (6·22 ± 0·03 km/s). The intercept on the time axis was 5·35 ± 0·02 s. With this number of points the errors have a considerable chance of themselves being wrong and they are therefore of little value. This velocity, assuming the error to be correct, lies, however, within limits which overlap those of the velocity obtained at station 2 from the five most distant points.

With the single observation which lies at a distance less than the three points providing the high-velocity line, it has been assumed that it would lie on a line of equal slope to that of the 16200 ft./s (4·94 km/s) line of the previous station. If this assumption is justifiable then the time intercept of this line is 4·43 s.

Wherever possible on the records readings have been taken of the times of sudden changes in amplitude occurring after the first arrivals, and before the direct sound. If these arrivals from all three stations are brought to a common depth of sea and then plotted together the scatter on the time-distance diagram is such as to preclude their all belonging to one or two particular phases. On the other hand, if they are plotted for each day separately there appears to be some justification for drawing straight lines through them. Those for station 1, for example, might be placed on a line of approximate slope equivalent to a velocity of 6600 ft./s (2·01 km/s) (figure 6). This velocity is less than the limiting sediment velocity obtained from the multiple refractions, and if therefore there is justification for drawing this line then it implies that within the region where there is a velocity gradient there is a layer of constant velocity producing these arrivals and that the straight line should be tangential to the time-distance curve of the multiple refractions. It cannot represent a layer lying below the point where the velocity gradient is zero, since no arrivals are possible from a layer in which the velocity is lower than in any layer lying above it.

For the second and third arrivals of station 2, two lines can be drawn which are tangential to the multiple refraction curve, but again the residuals are considerable. One of these lines has a slope equivalent to the velocity (8200 ft./s, 2·50 km/s) where the gradient becomes zero, and the second a slope equivalent to 6450 ft./s (1·97 km/s).

The second arrivals of station 3 lie on a line of smaller slope than the two low-velocity lines of stations 1 and 2, which is equivalent to a velocity of approximately 7200 ft./s (2·19 km/s); the residuals are no greater than would be expected from the accuracy with which the arrival times can be taken from the records.

An objection to the hypothesis that the second and third arrivals represent tracks through a layer of constant velocity within the region of the velocity gradient lies in the fact that the constant-velocity layer lying immediately below this region must be of great thickness compared with any layers of constant velocity which lie above it. This layer, however, does not produce good second arrivals. It is possible that the second arrivals of station 3 do represent the layer of constant velocity below the gradient region, the value of this velocity being less than that for station 2 in which, as has been shown above, the value has been determined with accuracy. An objection to this explanation is that the evidence from all the multiple refractions for the three stations points to the same gradient value in the upper layers and the consequent improbability that different asymptotic values of the velocity would be reached.

5·3. *The thickness of the layers*

The existence of a velocity gradient in layer 2 complicates the determination of the depths of interfaces lying below this layer. With a velocity-depth distribution such that the velocity in layer 2 increases linearly from a value equal to the velocity in layer 1 (i.e. the sea) to the value in layer 3 and such that all other layers have velocities which are independent of depth, the following relationships provide the thicknesses h_2, h_3 and h_4 in terms of quantities which can be obtained from the time-distance curves:

$$h_2 = \frac{V_3 - V_1}{a},$$

$$T_4 = \frac{2h_1}{V_1}\sqrt{\left(\frac{1-V_1^2}{V_4^2}\right)} + \frac{2h_3}{V_3}\sqrt{\left(1-\frac{V_3^2}{V_4^2}\right)} + \frac{2}{a}\left[\sinh^{-1}\left(\frac{\sqrt{(1-V_1^2/V_4^2)}}{V_1/V_4}\right) - \sinh^{-1}\left\{\frac{\sqrt{(1-V_3^2/V_4^2)}}{V_3/V_4}\right\} - \left\{\sqrt{\left(1-\frac{V_1^2}{V_4^2}\right)} - \sqrt{\left(1-\frac{V_3^2}{V_4^2}\right)}\right\}\right],$$

$$T_5 = \frac{2h_1}{V_1}\sqrt{\left(1-\frac{V_1^2}{V_5^2}\right)} + \frac{2h_3}{V_3}\sqrt{\left(1-\frac{V_3^2}{V_5^2}\right)} + \frac{2h_4}{V_4}\sqrt{\left(1-\frac{V_4^2}{V_5^2}\right)} + \frac{2}{a}\left[\sinh^{-1}\left\{\frac{\sqrt{(1-V_1^2/V_5^2)}}{V_1/V_5}\right\} - \sinh^{-1}\left\{\frac{\sqrt{(1-V_3{}^2/V_5{}^2)}}{V_3/V_5}\right\} - \left\{\sqrt{\left(\frac{1-V_1{}^2}{V_5{}^2}\right)} - \sqrt{\left(\frac{1-V_3{}^2}{V_5{}^2}\right)}\right\}\right],$$

where a is the velocity gradient in layer 2,

h_n is the thickness of layer n,
V_n is the velocity in layer n, and
T_n is the time intercept representing arrivals along layer n parallel to the interface.

The thicknesses of the various layers determined by the insertion of the experimental results into these expressions are shown in figure 15.

Except for station 2 the thicknesses shown have been obtained by assumptions concerning the similarities between the structures at the three positions. Thus, the limiting velocity below the region of continuously increasing velocity has been assumed to be the same for all three positions. The measurements obtained from stations 1 and 3 do not allow the calculation of this limiting velocity with an accuracy as great as that obtainable from station 2. The second assumption which has been made is that the velocity in the layer lying above that with velocity 20400 ft./s (6·22 km/s) on station 3 is the same as that of the analogous layer of station 2, namely, 16200 ft./s (4·94 km/s). In view of the fact that a layer of similar velocity was also found under station 1 this assumption is the most plausible.

It results, however, in the conclusion that the thickness of layer 1 is considerably less than at the other two stations. With the positions of the observations as obtained by the ship's navigating officers this conclusion is geologically difficult to interpret. As will be seen from figure 4 the shots for stations 1 and 3 are practically collinear, with the buoy positions of the latter station being close to the more distant shots of the former. No evidence for a component of dip in the direction of the line was obtained from either station 1 or station 2. If it had exceeded 0·7° it would have been detected in the experimental results of station 2 by the arrivals on the various buoys obtained from a single depth charge lying on a line of different slope from the best straight line fitted to the arrivals from the separate depth charges. This maximum angle of dip which could remain undetected could account for approximately 1000 ft. (300 m) difference in thickness between the positions of stations 1 and 3. The difference as calculated is approximately 3000 ft. (1000 m).

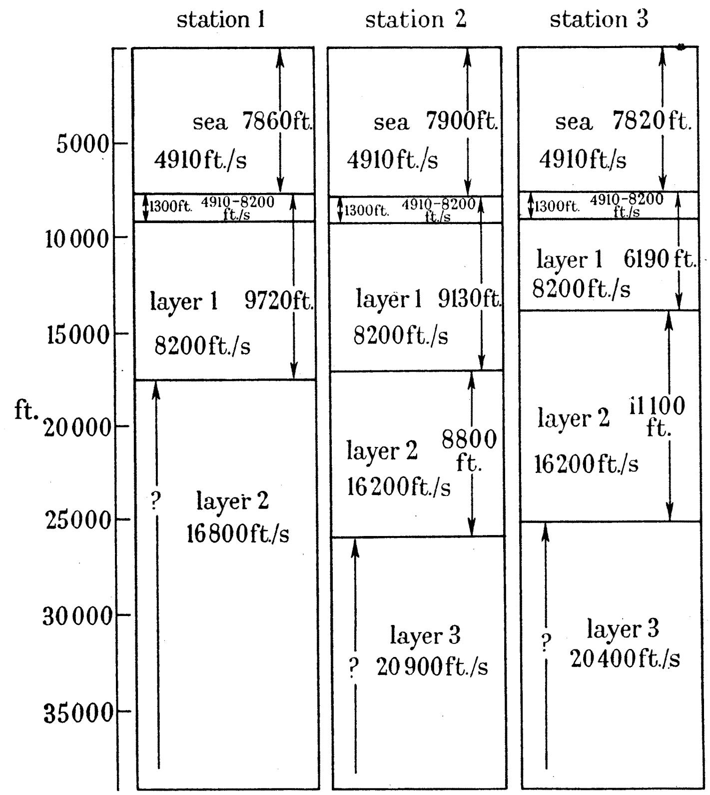

Figure 15. The thicknesses of the layers at the three stations.

It cannot be assumed that the component of dip for the observations at station 3 had a value sufficient to account for the calculated difference, since the shots would then have been fired 'up dip' and the apparent velocity of the line on which the single arrival lies would be higher than the assumed value of 16200 ft./s (4·9 km/s). This line would then pass below the closest arrival belonging to the deeper layer. For this same reason even if we postulated a change of material for this layer we could not select a velocity in it high enough to make the thickness of layer 1 as great as on the other occasions.

It must therefore be concluded that, if this 16200 ft./s (4·94 km/s) layer exists beneath station 3, it lies at a different depth, and that the change in depth must occur between the end of the line of station 1 and the buoy positions for station 3. There is some doubt as to the position of the line of station 1, and if the estimated position were shifted some miles to the south-east the explanation of the differences in depth would be easier. Thus, to the north-west of the area the water shoals to depths of 1000 fathoms (1800 m). It is probable that this shoaling is associated with a change in the depth to the base of the sediments which themselves might be thinner. The change in the depth of the water is about 350 fathoms (640 m), and it might be reasonable to assume that the sediment base changed in depth by an amount greater than this. In a direction at right angles to the direction in which the lines were shot there might therefore be a considerable component of dip.

6. The interpretation of the results

6·1. *The low-velocity layer*

The top layer of the seismic experiments consists of a part in which the velocity increases linearly with depth overlying another part in which the velocity is constant. The change in gradient from a finite value to zero may not be discontinuous. The starting velocity at the top of the layer is equal to the velocity of sound in water, and this indicates a high void ratio (i.e. the ratio of the volumes of liquid and solid) such as would be obtained from undisturbed and recent sediments. On account of the increase of stress with depth below the surface of this layer it would be expected that the void ratio would decrease and the longitudinal velocity increase to asymptotic values. This condition would not necessarily be that of zero void ratio. It is therefore possible to explain the velocity gradient in terms of the stress gradient, and it is not necessary to postulate that the material of this layer is other than of uniform origin and chemical composition throughout its thickness. The value of the gradient is greater than is found in terrigenous sediments. For example, Leet (1940) quotes values for coastal and land sedimentary columns which are not greater than approximately 1·3/s as against the value of 2·5/s found in our experiments.

This difference might be explained by the chemical composition or by the particle size. It has been shown (Skempton 1944) that the larger the median particle size the smaller will be the gradient of void ratio with depth.

Further, it has been shown by Faust (1951) that for sandstones and shales an increase in the time since deposition causes an increase in the velocity gradient at a given depth. This may explain the difference between our sedimentary column and that considered by Leet which was of Cretaceous-Tertiary age.

We have therefore no evidence from the seismic results to show that the sediments of the area have been of varying origin throughout their thickness. On the other hand, we have no positive evidence which could allow us to assert that this was indeed the true explanation. It might well be that there had been slow changes from continental to oceanic sedimentation or that there had been extensive slumping which would not be apparent from the depth-velocity relationship. We can, however, be confident in the suppositions that within this layer there are no major unconformities, and that throughout it consists of uncrystallized sediments.

The thickness and depths to the base of the sedimentary layer at the three stations are shown in table 9.

Table 9

station no.	total sediment thickness (ft.)	(m)	depth of base below sea-level (ft.)	(m)
1	9720	(2960)	17580	(5360)
2	9130	(2780)	17050	(5200)
3	6190	(1880)	14010	(4270)

It was not possible to obtain samples of the sea bed in this area, and for this reason there can be no certainty concerning the type of pelagic sedimentation at present occurring. It will not contain a high proportion of material derived directly from the present continental masses, since the surface current of the region is from the westward while the deep current, which is estimated to be of an inadequate velocity to have much eroding effect, is from the north. The oscillatory tidal currents cannot at present be effective in the transportation of terrigenous material from the existing land masses to this area, since the net transport can only be on the downward slopes from the continents on account of the small horizontal amplitudes of the currents, while a trough of deeper water lies between our area and the coast of Ireland.

To the south of the area where the water deepens to 1500 fm. (2700 m) and more there are indications on the Admiralty charts of globigerina ooze, but between the 1000 fm. (1800 m) and 1500 fm. (2700 m) contours the charts show mud, sand and clay. It is possible that the existence of the coarser material can be explained by transportation from the relatively shallow water area to the north-east.

The unconsolidated sedimentary layer thickness is greater than has been found by Ewing (1950) in the north-western Atlantic Ocean Basin. In a depth of 2800 fm. (5100 m) and with the assumption of a uniform velocity of 5600 ft./s (1·7 km/s) he calculates a thickness of 4500 ft. (1370 m). It would seem possible that this thickness should be increased because of the fact that no allowance has been made for the increase of velocity with depth.

If this layer does not represent undisturbed deep-sea sediments, then the most probable explanation of its origin is that it represents the seaward edge of a continental shelf shallowing to the north-eastwards which has been subsequently downwarped. The thickness is consistent with the seismic measurements made on the European (Bullard & Gaskell 1941) and eastern United States (Ewing *et al.* 1937, 1939, 1940, 1950*b*) continental shelves. Any attempts, through this hypothesis, to decide on the position of the continental mass providing the sediments will not be possible until further experimental results are available.

6·2. *The* 16000 *ft./s layer*

The thicknesses and depths below sea-level of the 16000 ft./s (4·9 km/s) layer on the two occasions when measurement was possible are shown in table 10.

Table 10

station no.	thickness of layer	depth of base below sea-level	velocity
1	—	—	16800 ± 300 ft./s (5·12 ± 0·09 km/s)
2	8800 ft. (2680 m)	25900 ft. (7890 m)	16200 ± 200 ft./s (4·94 ± 0·06 km/s)
3	11100 ft. (3380 m)	25200 ft. (7680 m)	16200 ft./s (assumed) (4·94 km/s)

The material of this layer cannot be identified solely from knowledge of the velocity of compressional waves in it, but at least we know it to be either a crystalline sedimentary rock or an igneous rock. The compressional-wave velocity in unconsolidated or uncrystallized sedimentary rocks cannot be as high as 16000 ft./s (4·9 km/s).

An important feature of this layer is that in spite of its variation in thickness between the sites of stations 2 and 3 its base is at roughly the same distance below sea-level. It is for this reason improbable that this layer is a sedimentary rock non-conformable with the layer above it, since it would then be necessary to postulate thicknesses of deposition differing by 2300 ft. (700 m) on a base which is apparently flat. Without more knowledge of this lower interface, however, the possibility cannot be ruled out. A more plausible interpretation of the nature of this rock is that it represents crystalline sediments which in origin form a continuous succession with the layer above, and that the differing levels of the top surface represent changes in the thickness to which the crystallization has extended.

If this layer is igneous in nature it is possibly a basaltic lava flow associated with the Tertiary flows of the northern British Isles, of the Rockall Bank and of Iceland, although the velocity of compressional waves in it is lower than might be expected on this basis. The only two determinations of the velocity in plateau basalt of which we are aware gave values of 18400 ft./s (5·61 km/s) and 16800 ft./s (5·12 km/s) (Geological Society of America, Special Paper no. 36), the latter figure being for a layer lying approximately 2500 ft. (760 m) deep. These determinations, even if representative, would have to be increased if applied to a layer at a depth of about 16000 ft. (4900 m).

6·3. *The* 20000 *ft./s layer*

The two velocity determinations in this layer were:

station 2 20900 ± 400 ft./s (6·37 km/s),
station 3 20400 ± 100 ft./s (6·23 km/s).

The thickness of this layer was not determined, but is certainly very great. It is of interest to decide whether this material is similar to the basement rocks occurring in the continents. The velocity of the latter is very variable, and large areas can be found giving results as low as 16800 ft./s (5·1 km/s) and as high as 20900 ft./s (6·38 km/s) (see, for example, Willmore 1949). Jeffreys (1947) gives a mean for the 'granitic layer' of 18400 ft./s (5·998 km/s). The velocity found in the present work is therefore consistent with the material being similar to that found beneath the continents, even when allowance is made for the effects of pressure. A layer with a velocity of 8·1 km/s (26800 ft./s) would have been detected in our work if it lay at a depth of less than 40000 ft. (12 km) below sea-level.

A number of workers on near earthquakes have found an 'intermediate layer' below the granitic layer and above the Mohorovičić discontinuity. The velocity in it is about 21300 ft./s (6·5 km/s), but its existence is not established beyond doubt. The properties of the layer found in the present work are consistent with its being similar to the 'intermediate layer'.

The results therefore show that the sediments in this part of the Atlantic are not underlain immediately by material similar to that below the Mohorovičić discontinuity. It is of great importance to determine how far this applies to the deeper parts of the North Atlantic. The velocities found by Ewing (1950) and Gaskell & Swallow (1951) are in general higher than those found in the present work, but still below 8·1 km/s. It may be that the basement rocks of the continent thin gradually westwards, and that at this point there is still a substantial thickness of rocks giving a velocity of 6·3 km/s, which are absent in the deeper part of the ocean. Some thinning of the continental rocks is, of course, necessary to avoid a large free-air gravity anomaly.

The experiments which are described above involved the assistance of many persons and organizations and to all these I am deeply indebted. It is impossible to provide a detailed acknowledgement to them all, and to those not specifically mentioned I offer my apologies.

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Of my collaborators in this work Mr J. C. Swallow bore a great share both from the practical and theoretical aspects, and but for his absence abroad would have jointly produced this paper. To him and to Mr J. C. Cleverly who accompanied me on every preliminary expedition as well as that in the *Weather Explorer* I am most grateful. Finally, I must not fail to mention the contribution to this work by Dr E. C. Bullard, F.R.S., who initiated the project and to Mr B. C. Browne who throughout provided most valuable stimulation, criticism and advice.

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